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The collective motion of self-propelled particles affected by the spatial-dependent noise based on Vicsek rules

Jia-xin Qian, Yan-qing Lu *

National Laboratory of Solid State Microstructures, Collaborative Innovation Center of Advanced Microstructures, College of Engineering and Applied Sciences, Nanjing University, Nanjing, 210093, China

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ABSTRACT

We study the collective motion of self-propelled particles affected by the spatial-dependent noise based on the Vicsek rules. In our model, we consider period boundary condition and only the particles inside a special region will be affected by noise. The consideration of the spatial-dependent noise is closer to reality because of the complexity of the environment. Interestingly, we find that the average deviation between the average motional direction of the system and the orientation of the noisy region is very close to zero when the amplitude of noise is large enough. Particular orientation of the noisy region makes the motional direction of the system parallel to the orientation of the noisy region. The adjustment of the motional direction of the system also depends on the shape, the proportion and the spatial distribution of the noisy region. Our findings may inspire the capture of the key features of collective motion underlying various phenomena.

1. Introduction

Collective behaviors of active agents are universally observed in the systems with rich scales from macroscopic to microscopic including the crowds of human [1], the schools of fish [2] and the colonies of bacteria [3] etc [4]. Because of the complexity of the environment, most of the time, the behavior of agents will inevitably encounter noise. Noise is present in the study of many domains [5–13] and it plays an important role on the dynamics of the system including the system of active matter [14–16]. Studying how different kinds of noise affect the collective motion and what the noise can do in the system of active matter is useful in exploring the basic principles of collective motion, taking advantage of the features of noise and avoiding the unexpected disturbance caused by noise.

One of the typical models to studying collective motion, named Vicsek model, is proposed by Vicsek et al. in 1995 [17]. In Vicsek model, all of the self-propelled particles follow the rule of velocity alignment which considers both the average velocity of the neighbor of particles and normal-distributed random noise to update the velocity of the particles [17,18]. Following Vicsek et al. many researchers show interest in studying the collective motion with different kinds of noise including cross-correlated noise [19–22], non-Gaussian noise [23], colored noise [24,25], Telegraphic-like noise [26] and hybrid noise [27] etc [28]. And many exotic phenomena resulting from noise are found. Noise can induce the transition of the motional state of the system [29–31] and lead to symmetry breaking [32]. Driving the particles [33,34] and maximizes collective motion in heterogeneous media are achieved by noise [35]. Noise also affects the criticality [36], synchronization [37] and the diversity of collective motion in Vicsek model [38].

* Corresponding author.
E-mail address: yqlu@nju.edu.cn (Y.-q. Lu).

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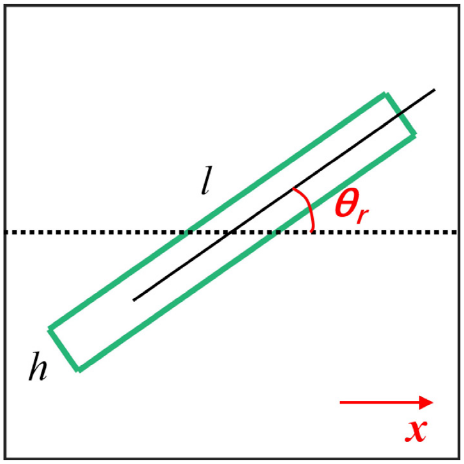

Fig. 1. The schematic diagram of our model with spatial-dependent noise. l and h denote the length and width of the noisy region respectively. And the orientation of the noisy region is denoted by the angle θ_r.

Although the studies on collective motion with noise make great progress in the recent decades, just a few studies consider the effect of the spatial feature of noise on collective motion [39]. Studying the collective motion with noise distributed nonuniform in space is an important step to further understanding the key factor underlying the various phenomena of collective motion. Therefore, we pay attention to the role of the spatial-dependent noise in the collective motion based on the Vicsek rules. The update of the velocity of the particles will be affected by noise just when the particles are inside the special region. By investigating the effect of the amplitude of noise, the orientation, the shape, the proportion and the spatial distribution of the noisy region on the average motional direction of the system, we find that the proper spatial-dependent noise can perfectly make the motional direction of the system parallel to the orientation of the noisy region.

2. Model and method

We consider N particles move in the complex environment which consists of infinite repeated units with noisy regions shown in Fig. 1. The rectangular region in green is a noisy region. We adapt periodic boundary condition in order to improve the efficiency of the simulations so that we can approximately evaluate the features of motion of the system just by simulating the motion of the system in one square cell shown in Fig. 1. The linear size of the square cell is L and the particles are regarded as points. Initially, particles are randomly distributed in the cell. The position of all the particles update simultaneously at each time step Δt following

$$\mathbf{x_i}(t+\Delta t) = \mathbf{x_i}(t) + \mathbf{v_i}(t)\Delta t \tag{1}$$

where $\mathbf{x_i}(t)$ and $\mathbf{v_i}(t)$ denote the position and velocity of the particles i (i takes 1 to N) at time t respectively. The initial direction of velocity of the particles are distributed in $[-\pi, \pi]$ randomly and uniformly. The magnitude of velocity of each particle is v. Based on the rules of velocity alignment in Vicsek model, the update of the direction of velocity is as follows

$$\theta_i(t+\Delta t) = Arg[\sum_{j\in\mathcal{N}_i(t)} e^{i\theta_j(t)}] + \Delta\theta(t) \tag{2}$$

$\theta_i(t)$ denotes the direction of the velocity of particle i at time t. $\mathcal{N}_i(t)$ is the set of neighbors of the particle i which means $\mathcal{N}_i(t) = \{j : |\mathbf{x_i} - \mathbf{x_j}| \leqslant r\}$ and r is the interaction radius of each particle. $\Delta\theta$ denotes the noise.

Considering the effect of complex environment on motion, we take the spatial-dependent noise into account, which means

$$\Delta\theta(t) = \eta(\mathbf{x_i}(t))\xi_i(t) \tag{3}$$

where $\xi_i(t)$ is a random number uniformly distributed in $[-1/2, 1/2]$. When particle i is in the noisy region (the rectangular region in green shown in Fig. 1), $\eta(\mathbf{x_i}(t)) = \eta_n$ where η_n is the amplitude of noise inside the noisy region. Otherwise, $\eta(\mathbf{x_i}(t)) = 0$. As Fig. 1 shows, the orientation of the rectangular region is described by the angle θ_r between the long axis of the region and the positive direction of the X-axis which is shown by the red arrow. The length and width of the rectangular region are l and h respectively.

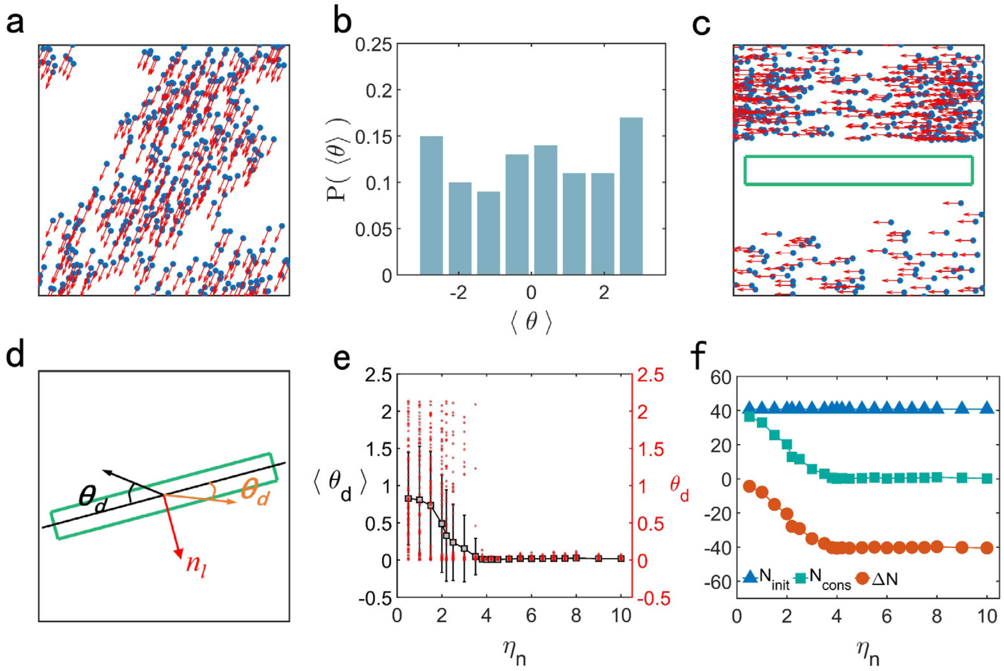

Fig. 2. The effect of the amplitude of the spatial-dependent noise on the adjustment of motional direction of the system. (a) the snapshot of all the particles move in almost the same direction without noise(η_n); (b) the probability distribution of the average motional direction $\langle\theta\rangle$ without noise; (c) the snapshot of the system moves with the effect of spatial-dependent noise (the rectangular region in green is the noisy region); (d) the schematic diagram of the deviation θ_d between the average motional direction of the system $\langle\theta\rangle$ and the orientation of the noisy region; (e) the average deviation $\langle\theta_d\rangle$, denoted by the black square points with gray shadow(with error bar), as a function of the amplitude of noise η_n. The red star points denotes the distribution of value of θ_d among 100 realizations for each fixed η_n.; (f) the number of the particles in the noisy region at the beginning of the simulation N_{init}, reaching motional consensus N_{cons} and the different $\Delta N = N_{cons} - N_{init}$ as a function of η_n. The value of other parameters for simulations are $l = 9, h = 10/9, r = 1.0$.

To characterize the feature of the collective behavior, normalized average velocity is introduced as the order parameter, which is

$$\phi = \frac{1}{Nv}\left|\sum_{i=1}^{N}\mathbf{v_i}\right| \tag{4}$$

3. Result and discussion

In the simulation, $\Delta t = 1, L = 10, v = 0.04$. When $\eta_n = 0$, the system will finally arrive at a state that all the particles move in the same direction as shown in Fig. 2(a). Fig. 2(b) shows the probability distribution of the average direction of *100* realizations when $\eta_n = 0$, which implies the motional direction of the system in each realization is random. In the present of the spatial-dependent noise, $\eta_n \neq 0$. We observe that the motional direction is parallel to the orientation of the rectangular region as shown in Fig. 2(c).

In order to know why the spatial-dependent noise can adjust the direction of the system, we first investigate the effect of the amplitude of noise η_n on the motion of the system. To measure the deviation between the average motional direction $\langle\theta\rangle = \sum_{i=1}^{N}\theta_i$ and the orientation of the rectangular region with noise θ_r, we consider the angle θ_d. θ_d is the smaller angle of the difference between the average direction of the velocity $\langle\theta\rangle$ and the orientation of the noisy region θ_r. The range of the value of θ_d is $[0, 2\pi]$.

In order to have a uniform and simple expression for θ_d in mathematics, we consider the normal vector $\mathbf{n_l}$ which is perpendicular to the length of the rectangular region. If the angle between the average motional direction $\langle\theta\rangle$ and the normal vector $\mathbf{n}_l$ is larger than 0.5π, the deviation is $\theta_d = \langle\theta\rangle - 0.5\pi$ as the black angle θ_d shown in Fig. 2(d). Otherwise, $\theta_d = 0.5\pi - \langle\theta\rangle$ as the orange angle θ_d shown in Fig. 2(d). That means

$$\theta_d = \begin{cases} \langle\theta\rangle - 0.5\pi & \langle\mathbf{v}\rangle\cdot\mathbf{n_l} < 0 \\ 0.5\pi - \langle\theta\rangle & \langle\mathbf{v}\rangle\cdot\mathbf{n_l} \geqslant 0 \end{cases} \tag{5}$$

where $\langle\mathbf{v}\rangle$ is the average velocity of the system.

As Fig. 2(e) shows, the average deviation between the average motional direction and the orientation of the noisy region decreases as the amplitude of noise η_n increases. When $\eta_n > 3.8$, the values of θ_d are very close to *0* and the values of θ_d in each simulation is very close to the orientation of the noisy region (as shown in Fig. 2e), which means good adjustment of motional direction of the system.

To understand how the spatial-dependent noise can adjust the motional consensus of the system, we study the number of particles in the noisy region in different stages of the simulations. As shown in Fig. 2(f), with the increasing of η_n, the number of the particles in the noisy region when the system reaches motional consensus N_{cons} decrease. And N_{cons} almost reaches *0* when $\eta_n > 3.8$. So do the different $\Delta N = N_{cons} - N_{init}$. That means all the particles will almost move out from the noisy region when the amplitude of spatial-dependent noise is larger than 3.8. And the average motional direction of the system will gradually parallel to the orientation of the noisy region.

At large noise amplitude, the difference between the motional direction of each particle moves into the noisy region and that moves out of the noisy region is large. Particles move out from the noisy region will reach motional consensus again but the average motional direction of the particles have large difference from the last-time of that they reach motional consensus. Particles will move into the noisy region and move out from the noisy region but with various average motional direction unless the average motional direction of the particles is parallel to the orientation of the noisy region. Based on the analysis above, the physical original is that the coordination between the construction of motional consensus outside a noisy region and the destruction of motional consensus inside the noisy region. When the amplitude of the noise outside the noisy region is small, the effect of η_n on the collective motion of the system will have the similar phenomena and rules as Fig. 2(e), (f) show.

Beside the amplitude of the noise, the orientation of the noisy region is also important to the adjustment of the average motional direction. We respectively investigate the average deviation of the motional direction $\langle\theta_d\rangle$ and $P(\theta_d < 0.03)$ which is the probability that θ_d is smaller than 0.03. As shown in Fig. 3(a) and (b), the average deviation of the motional direction is smaller than *0.4* with various orientations of noisy region. And the system moves parallel to the orientation of noisy region when $\theta r = 0, 0.25\pi, 0.5\pi, 0.75\pi, \pi$.

Although the average order parameter shown in Fig. 3(d) is large enough to denote the motional consensus of the system, the average direction of the system is not always parallel to the orientation of the noisy region. For example, when $\theta_r = 0.083\pi$ and $\theta_r = 0.583\pi$, the average motional direction of the system is horizontal and vertical respectively, as the probability distribution of the average motional direction shown in Fig. 3(c) and Fig. 3(e) respectively. The system will not prefer to move parallel to the orientation of the noisy region except $\theta r = 0, 0.25\pi, 0.5\pi, 0.75\pi, \pi$. Because of the periodic boundary condition, it will make particles move inside the noisy region as Fig. 3(f) shows. And the system will be adjusted to move in the direction that the particles will not move inside the noisy region.

The shape of the noisy region also has an impact on the motional direction of the system. Keeping the same proportion of the noisy region as $p = 0.1$, we change the length l of the rectangular noisy region to investigate the effect of shape on the motional direction. The length of the rectangular noisy region in Fig. 4(a), (c) and (e) are $l = 8.0$, $l = 5.0$ and $l = 3.0$ respectively. With the decreasing of the length, the noisy region gradually changes from a rectangle to a square. The motional direction of the system is from horizontal to both vertical and horizontal. When $l = 5.0$, most of the motional direction is horizontal and a few motional directions are vertical because of the available space for the system to move vertically outside the noisy region.

Considering the proportion of the noisy region affect the degree of adjustment of the motional direction, we study the effect of proportion of the noisy region by investigating the probability of realizations when the ratio of the particles moving horizontally s is larger than *95* percent. As shown in Fig. 5(a), the noisy region can not adjust the motional direction of the system at all when the proportion of the noisy region is larger than *0.7*. And the degree of adjustment to the motional direction decreases as p increase. Large interaction radius, such as $r = 2.0$, weakens the adjustment on motional direction which makes the adjustment unstable as shown in Fig. 5(b).

As for the different spatial distribution of the noisy region, we set two equal-proportion noisy regions in the square cell and study the adjustment of motional direction of the system. Both one noisy region and two noisy regions, with the same proportion, can adjust the motional direction of the system as shown in Fig. 6(a) and (b), but the degree of adjustment is different.

As Fig. 7(a) shows, the two noisy region can hardly adjust the motional direction of the system when p is larger than *0.6*. This value is smaller than that when it is just one noisy region, which implies the weaker ability to adjust the motional direction of the system compared to one noisy region with the same proportion. The difference of the ability to adjust the motional direction for different interaction radius is larger than that with one noisy region as shown in Fig. 7(b).

4. Conclusion

In conclusion, we study the collective motion of self-propelled particles affected by spatial-dependent noise based on Vicsek rules. The motion of the particles inside the rectangular region of the square cell will be affected by noise. While other particles will move without noise.

Our investigation reveals that spatial-dependent noise enables to adjust the average motional direction of the system rather than moving in the random direction. And the amplitude of noise, orientation, shape, proportion and spatial-distribution of noisy region have different impacts on the adjustment of the average motional direction.

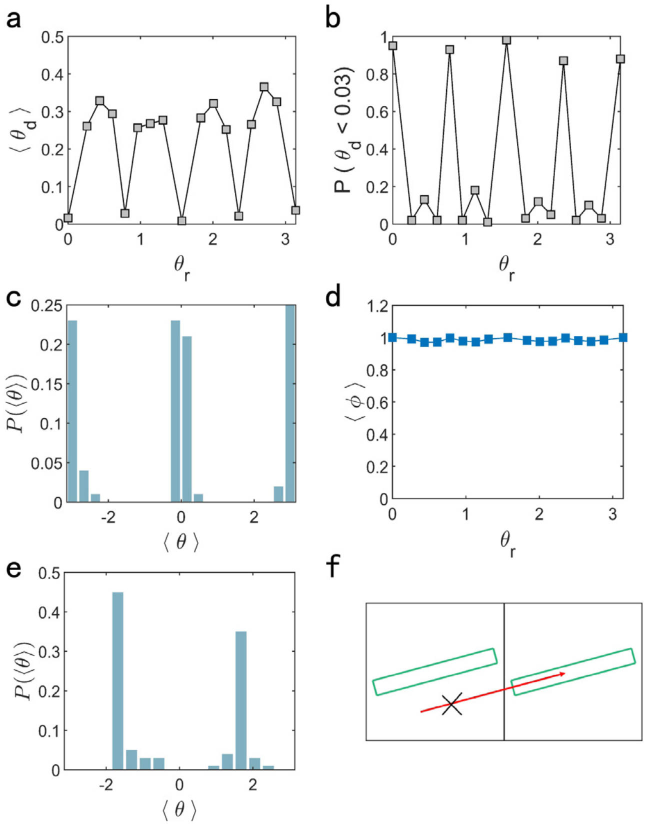

Fig. 3. The effect of the orientation of the noisy region on the adjustment of the motional direction of the system. (a) the deviation θ_d as a function of the orientation of the noisy region θ_r; (b) the probability that θ_d is smaller than 0.03 as a function of θ_r; The probability distribution of the average motional direction $\langle\theta\rangle$ when $\theta_r = 0.083\pi$(c) and $\theta_r = 0.583\pi$(e). (d) the average order parameter $\langle\phi\rangle$ as a function of θ_r; (f) the schematic diagram of the average motional direction is not parallel to the orientation of the noisy region because off the avoidance to move into the noisy region. The value of other parameters for simulations are $l = 9, h = 10/9, r = 1.0$.

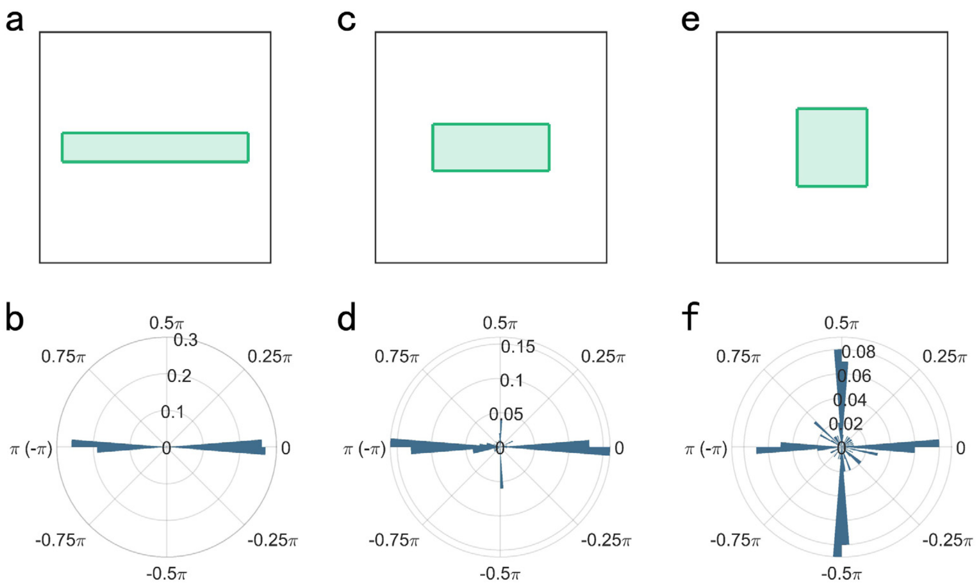

Fig. 4. The effect of the shape of the noisy region on the adjustment of the motional direction of the system. The schematic diagram of the rectangular noisy region with length $l = 8.0$(a), $l = 5.0$(c) and $l = 3.0$(e). The polardiagram of the probability distribution of the average motional direction $\langle\theta\rangle$ when the length of the noisy region is $l = 8.0$(b), $l = 5.0$(d) and $l = 3.0$(f). The interaction radius for simulations is $r = 1.0$.

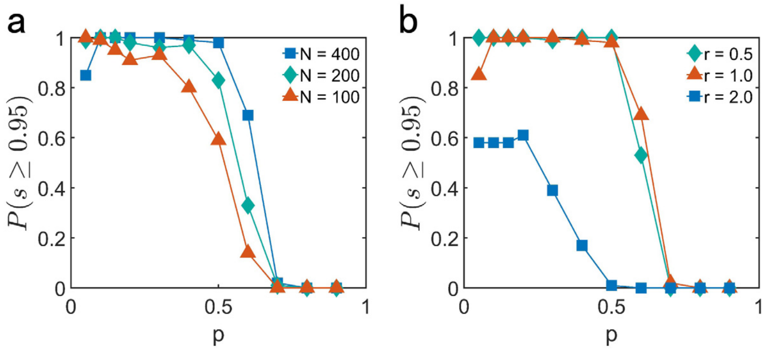

Fig. 5. The effect of the proportion of the noisy region on the adjustment of the motional direction of the system. The probability that the ratio of particles which move parallel to the orientation of the noisy region is larger than *95* percent $P(s \geqslant 0.95)$ as a function of the proportion of the noisy region p with different total number of particles(a) and different interaction radius(b). The value of other parameters for simulations are $l = 10, h = 1$.

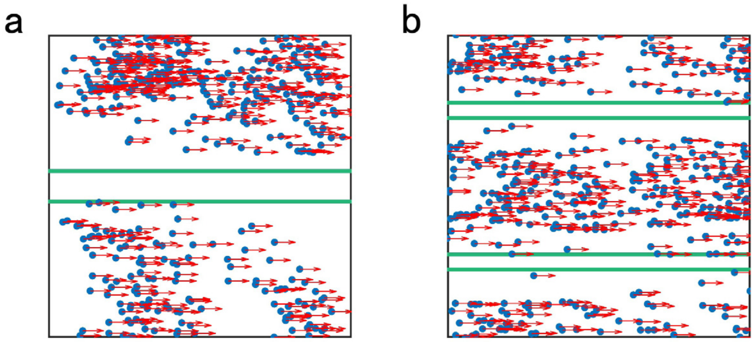

Fig. 6. The snapshots of the particles move in the square with one noisy region(a) and two noisy regions(b). Total proportion of the noisy region in (a) and (b) is *0.1*. The value of other parameters for simulations are $l = 10, h = 1$.

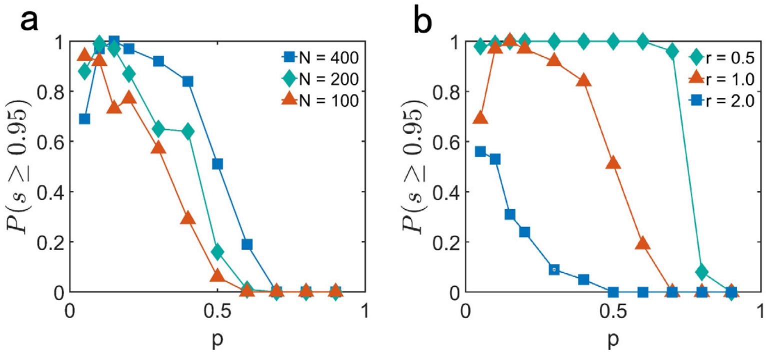

Fig. 7. The effect of total proportion of the two noisy region on the adjustment of the motional direction of the system. The degree of adjustment $P(s \geqslant 0.95)$ as a function of p with different total number of particles(a) and different interaction radius(b). The value of other parameters for simulations are $l = 10, h = 0.5$.

When the amplitude of noise is larger than *3.8*, all the particles move parallel to the orientation of the noisy region. When the orientation of the rectangular noisy region are $\theta_r = 0, 0.25\pi, 0.5\pi, 0.75\pi, \pi$, the average motional direction of the system is parallel to the orientation of the noisy region, which means the spatial-dependent noise controls the motional direction of the system. The motional direction of the system from just horizontal to both vertical and horizontal with the change in the shape of the noisy region from rectangle to square.

The degree of the adjustment of the motional direction decreases as the proportion of the noisy region increases both for one noisy region and two noisy regions. Large interaction radius weaken the degree of adjustment of the motional direction. The difference of adjustment between different r is larger with two noisy regions although the total proportion of the noisy region is equal.

Our research may encourage further studies on the collective behavior in various and complex environments. Building the complex environment by considering spatial-dependent noise can be used in various model, including ABPs and RTPs, to study the phenomena and rules of collective motion of self-propelled particles. And it is possible to control the motion of systems by changing the configuration of environment.

CRediT authorship contribution statement

Jia-xin Qian: Writing – original draft, Formal analysis, Data curation, Visualization, Methodology. **Yan-qing Lu:** Writing – review & editing, Supervision, Project administration, Funding acquisition.

Declaration of competing interest

The authors declared no potential conflicts of interest with respect to the research, author-ship, and publication of this article.

Data availability

Data will be made available on request.

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References

[1] N. Bain, D. Bartolo, Dynamic response and hydrodynamics of polarized crowds, Science 363 (6422) (2019) http://dx.doi.org/10.1126/science.aat9891.
[2] S. Hubbard, P. Babak, S.T. Sigurdsson, K.G. Magnússon, A model of the formation of fish schools and migrations of fish, Ecol. Model. 174 (4) (2004) 359–374, http://dx.doi.org/10.1016/j.ecolmodel.2003.06.006.
[3] A. Sokolov, I.S. Aranson, J.O. Kessler, R.E. Goldstein, Concentration dependence of the collective dynamics of swimming bacteria, Phys. Rev. Lett. 98 (15) (2007) 158102, http://dx.doi.org/10.1103/PhysRevLett.98.158102, URL https://www.ncbi.nlm.nih.gov/pubmed/17501387.
[4] T. Vicsek, A. Zafeiris, Collective motion, Phys. Rep. 517 (3–4) (2012) 71–140, http://dx.doi.org/10.1016/j.physrep.2012.03.004.
[5] F. Sagués, J.M. Sancho, J. García-Ojalvo, Spatiotemporal order out of noise, Rev. Modern Phys. 79 (3) (2007) 829–882, http://dx.doi.org/10.1103/RevModPhys.79.829.
[6] W. Su, J. Guo, X. Chen, G. Chen, Noise-induced synchronization of Hegselmann–Krause dynamics in full space, IEEE Trans. Automat. Control 64 (9) (2019) 3804–3808, http://dx.doi.org/10.1109/tac.2018.2885090.
[7] P.D. Pra, G. Giacomin, D. Regoli, Noise-induced periodicity: Some stochastic models for complex biological systems, in: Mathematical Models and Methods for Planet Earth, in: Springer INdAM Series, Springer, 2014, pp. 25–35, http://dx.doi.org/10.1007/978-3-319-02657-2_3.
[8] T. Shinbrot, F. Muzzio, Noise to order, Nature 410 (6825) (2001) 251–258, http://dx.doi.org/10.1038/35065689.
[9] W. Su, G. Chen, Y. Hong, Noise leads to quasi-consensus of Hegselmann–Krause opinion dynamics, Automatica 85 (2017) 448–454, http://dx.doi.org/10.1016/j.automatica.2017.08.008.
[10] K. Matsumoto, I. Tsuda, Noise induced order, J. Stat. Mech. Phys. 31 (1) (1983) 87–106, http://dx.doi.org/10.1007/BF01010923.
[11] L.S. Tsimring, Noise in biology, Rep. Progr. Phys. 77 (2) (2014) 026601, http://dx.doi.org/10.1088/0034-4885/77/2/026601, URL https://www.ncbi.nlm.nih.gov/pubmed/24444693.
[12] C. Boettiger, From noise to knowledge: how randomness generates novel phenomena and reveals information, Ecol. Lett. 21 (8) (2018) 1255–1267, http://dx.doi.org/10.1111/ele.13085, URL https://www.ncbi.nlm.nih.gov/pubmed/29790295.
[13] W. Su, Y. Yu, Free information flow benefits truth seeking, J. Syst. Sci. Complexity 31 (4) (2017) 964–974, http://dx.doi.org/10.1007/s11424-017-7078-4.
[14] J. Jhawar, R.G. Morris, U.R. Amith-Kumar, M. Danny Raj, T. Rogers, H. Rajendran, V. Guttal, Noise-induced schooling of fish, Nat. Phys. 16 (4) (2020) 488–493, http://dx.doi.org/10.1038/s41567-020-0787-y.
[15] M. Kosuke, J.J. Molina, Noise induced phase separation in active systems: Creating patterns with noise, 2018, arxiv.
[16] D.S. Cambui, T. Iliass, Behavioral transitions induced by speed and noise in animal aggregates, Internat. J. Modern Phys. B 31 (11) (2017) http://dx.doi.org/10.1142/s0217979217500734.
[17] T. Vicsek, A. Czirok, E. Ben-Jacob, I.I. Cohen, O. Shochet, Novel type of phase transition in a system of self-driven particles, Phys. Rev. Lett. 75 (6) (1995) 1226–1229, http://dx.doi.org/10.1103/PhysRevLett.75.1226, URL https://www.ncbi.nlm.nih.gov/pubmed/10060237.
[18] F. Ginelli, The physics of the Vicsek model, Eur. Phys. J. Spec. Top. 225 (11–12) (2016) 2099–2117, http://dx.doi.org/10.1140/epjst/e2016-60066-8.
[19] D. Wu, S. Zhu, Effects of cross-correlated noises on the transport of active Brownian particles, Phys. Rev. E 90 (1) (2014) 012131, http://dx.doi.org/10.1103/PhysRevE.90.012131, URL https://www.ncbi.nlm.nih.gov/pubmed/25122275.
[20] C. Zheng, R. Toenjes, Noise-induced swarming of active particles, Phys. Rev. E 106 (6) (2022) http://dx.doi.org/10.1103/PhysRevE.106.064601.
[21] Z.-L. Jia, C.-Y. Yang, B.-Y. Ma, Y. Chen, Noise enhanced stability of an active particle in a spatial metastable potential driven by cross-correlated noises, J. Stat. Mech. Theory Exp. 2022 (6) (2022) http://dx.doi.org/10.1088/1742-5468/ac7792.
[22] Y. Fang, Y. Luo, T. Huang, C. Zeng, Ratchet effect of interacting active particles induced by cross-correlated noises, Eur. Phys. J. B 95 (5) (2022) http://dx.doi.org/10.1140/epjb/s10051-022-00335-8.
[23] E. Fodor, H. Hayakawa, J. Tailleur, F. van Wijland, Non-Gaussian noise without memory in active matter, Phys. Rev. E 98 (6) (2018) http://dx.doi.org/10.1103/PhysRevE.98.062610.

[24] R. Wittmann, J.M. Brader, A. Sharma, U.M.B. Marconi, Effective equilibrium states in mixtures of active particles driven by colored noise, Phys. Rev. E 97 (1–1) (2018) 012601, http://dx.doi.org/10.1103/PhysRevE.97.012601, URL https://www.ncbi.nlm.nih.gov/pubmed/29448463.
[25] C. Maggi, N. Gnan, M. Paoluzzi, E. Zaccarelli, A. Crisanti, Critical active dynamics is captured by a colored-noise driven field theory, Commun. Phys. 5 (1) (2022) http://dx.doi.org/10.1038/s42005-022-00830-5.
[26] J. Um, T. Song, J.-H. Jeon, Langevin dynamics driven by a telegraphic active noise, Front. Phys. 7 (2019) http://dx.doi.org/10.3389/fphy.2019.00143.
[27] D.S. Cambui, Vicsek model of self-propelled particles with hybrid noise, Modern Phys. Lett. B 34 (14) (2020) http://dx.doi.org/10.1142/s0217984920501444.
[28] K. Cheng, P. Liu, M. Yang, M. Hou, Experimental investigation of active noise on a rotor in an active granular bath, Soft Matter 18 (13) (2022) 2541–2548, http://dx.doi.org/10.1039/d1sm01798e, URL https://www.ncbi.nlm.nih.gov/pubmed/35166750.
[29] A. Mikhailov, D. Zanette, Noise-induced breakdown of coherent collective motion in swarms, Phys. Rev. E 60 (4) (1999) 4571–4575, http://dx.doi.org/10.1103/PhysRevE.60.4571.
[30] U. Erdmann, W. Ebeling, A.S. Mikhailov, Noise-induced transition from translational to rotational motion of swarms, Phys. Rev. E 71 (5 Pt 1) (2005) 051904, http://dx.doi.org/10.1103/PhysRevE.71.051904, URL https://www.ncbi.nlm.nih.gov/pubmed/16089568.
[31] K.S. Sergeev, T.E. Vadivasova, A.P. Chetverikov, Noise-induced transition in a small ensemble of active Brownian particles, Tech. Phys. Lett. 40 (11) (2014) 976–979, http://dx.doi.org/10.1134/s1063785014110108.
[32] R. Rafeek, D. Mondal, Noise-induced symmetry breaking of self-regulators: Nonequilibrium transition toward homochirality, J. Chem. Phys. 154 (24) (2021) 244906, http://dx.doi.org/10.1063/5.0053856, URL https://www.ncbi.nlm.nih.gov/pubmed/34241341.
[33] S. Klumpp, A. Mielke, C. Wald, Noise-induced transport of two coupled particles, Phys. Rev. E 63 (3 Pt 1) (2001) 031914, http://dx.doi.org/10.1103/PhysRevE.63.031914, URL https://www.ncbi.nlm.nih.gov/pubmed/11308685.
[34] D. Breoni, H. Lowen, R. Blossey, Active noise-driven particles under space-dependent friction in one dimension, Phys. Rev. E 103 (5–1) (2021) 052602, http://dx.doi.org/10.1103/PhysRevE.103.052602, URL https://www.ncbi.nlm.nih.gov/pubmed/34134234.
[35] O. Chepizhko, E.G. Altmann, F. Peruani, Optimal noise maximizes collective motion in heterogeneous media, Phys. Rev. Lett. 110 (23) (2013) 238101, http://dx.doi.org/10.1103/PhysRevLett.110.238101, URL https://www.ncbi.nlm.nih.gov/pubmed/25167531.
[36] G. Baglietto, E.V. Albano, J. Candia, Criticality and the onset of ordering in the standard Vicsek model, Interface Focus 2 (6) (2012) 708–714, http://dx.doi.org/10.1098/rsfs.2012.0021, URL https://www.ncbi.nlm.nih.gov/pubmed/24312724.
[37] N. Bruot, L. Damet, J. Kotar, P. Cicuta, M.C. Lagomarsino, Noise and synchronization of a single active colloid, Phys. Rev. Lett. 107 (9) (2011) 094101, http://dx.doi.org/10.1103/PhysRevLett.107.094101, URL https://www.ncbi.nlm.nih.gov/pubmed/21929245.
[38] G. Chen, Small noise may diversify collective motion in Vicsek model, IEEE Trans. Automat. Control 62 (2) (2017) 636–651, http://dx.doi.org/10.1109/tac.2016.2560144.
[39] B.-Q. Zhang, Z.-G. Shao, Collective motion of self-propelled particles with complex noise environments, Physica A 563 (2021) http://dx.doi.org/10.1016/j.physa.2020.125382.