Mathematical model for manufacturing microfiber coil resonators

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Abstract. We present a mathematical model exemplifying the manufacture of microfiber coil resonators (MCRs) with rotational and translational stages controlled by a computer. The MCR profiles are related to the stage positions; the result is important for practical manufacturing and application of MCRs. [DOI: 10.1117/1.3377988]

Subject terms: microfiber; microresonator; manufacture.

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1 Introduction

Evanescent-field-based optical resonators in the form of microspheres, photonic crystals, and microrings have been under intensive investigation. For these applications, small size, low loss, and high Q-factor are the dominant requirements. The drawback that all high-Q resonators present relates to the difficulty of coupling light into and out of the resonator. Microfiber-based resonators have recently attracted much attention because of the enormous progress in the fabrication of low-loss submicrometric optical wires, which allow for low-loss evanescent guiding.[1–3] Moreover, optical microresonators fabricated from subwavelength microfibers offer several prospective benefits a compared to other kinds of microresonators; these include low insertion loss, complete fiber compatibility, and flexibility. As a basic functional element of the microfiber photonic circuit, the microfiber resonator (MR) has increasingly attracted interest.[4–10] Being fabricated of a single-mode fiber, the MR does not have the problem of coupling in and out of the resonator with the ends of standard sizes. In addition, as opposed to the microresonators fabricated lithographically, the MCR does not suffer from surface roughness introduced by etching, and may eventually exhibit the smallest loss. Two types of 2-D MR have been reported in previous publications: the self-touching loop resonator and the knot resonator.[11,12] A 3-D multiple-turn MR, the microfiber coil resonator (MCR), was suggested in 2004[4] and demonstrated in the following years by wrapping the microfiber on a low index rod in air[7,8] and liquid,[5] and was coated with Teflon.[8] Its appeal resides in its higher flexibility, ultrahigh Q-factor, and the possibility to be employed as a high sensitivity microfludic sensor,[13,14] a slow light generator, an optical signal processor, and much more. All applications are still seriously limited because of the manufacturing technique. Until now, the profile of manufactured MCRs is still uncontrollable. Ideally, the setup—including rotational and translational stages—should produce a MCR with accurate predesigned profiles. In this work, we theoretically investigated the mathematical model that relates the MCR profiles to the manufacturing setup, which is important for manufacturing MCRs with predesigned profiles in future applications.

2 Mathematical Model

It is desirable to manufacture the MCR with a set of controllable precision stages, including a rotation and several translational stages. A general schematic of the process is shown in Fig. 1, where the dark cylinder is the support rod and PA is the microfiber to be wrapped; while one extremity (A) is fixed, the other one (P) is moved by the rotation and translational stages. The wrapping process is simulated, assuming that Q (which represents the point of contact between the microfiber and the support rod) moves with P. The support rod is supposed to be static. By thinking about the relative position between P and the rod, the model is also suitable for the case of using a rotating rod with a simply translational stage, which is easier in practical fabrication. A cylindrical coordinates system is used and

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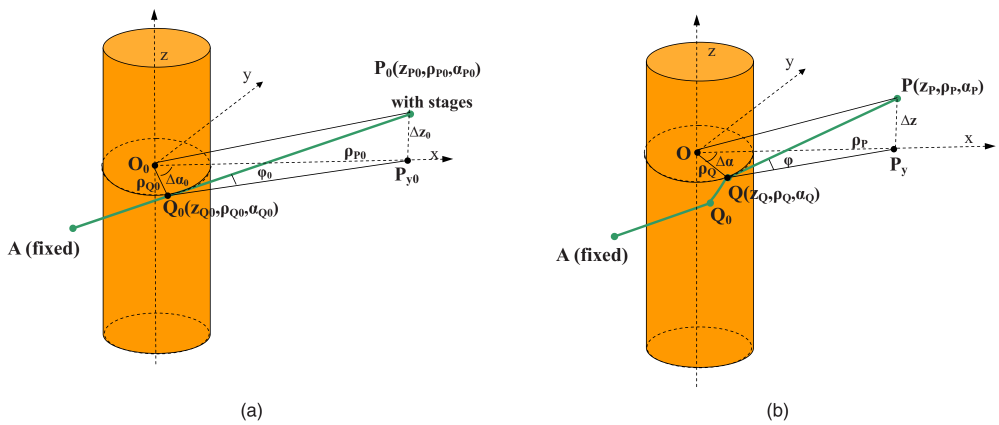

Fig. 1 Mathematical model of manufacturing 3-D MCRs. (a) and (b) represent the relative positions of the microfiber with respect to the support rod at time 0 and t, respectively.

$P(z_P,\rho_P,\alpha_P)$, $Q(z_Q,\rho_Q,\alpha_Q)$, z_P,ρ_P,α_P, z_Q,ρ_Q,α_Q are assumed to be functions of time t. Figure 1(a) shows the initial status at time 0; Fig. 1(b) shows the status at time t, where $OP_y=\rho_P$, $OQ=\rho_Q$, $\Delta z=PP_y$, $\Delta\alpha=\angle QOP_y$, $\varphi=\angle PQP_y$, and the initial position at time 0 is $P_0(z_{P0},\rho_{P0},\alpha_{P0})$, $Q_0(z_{Q0},\rho_{Q0},\alpha_{Q0})$, $\Delta\alpha_0$, Δz_0, and φ_0.

Simple geometrical considerations allow us to write the following relations:

$$\cos\Delta\alpha=\rho_Q/\rho_P, \tag{1}$$

$$\tan\varphi=\frac{dz_Q}{\rho_Q d\alpha_Q}=\frac{\Delta z}{\sqrt{\rho_P^2-\rho_Q^2}}, \tag{2}$$

$$\Delta\alpha=\alpha_P-\alpha_Q, \tag{3}$$

$$\Delta z=z_P-z_Q. \tag{4}$$

Generally ρ_Q can be assumed constant, and $\alpha_P(t)=\omega t+\alpha_{P0}$ (where ω is P angular velocity). For a given desired MCR geometry (Q position), $z_Q(\alpha_Q)$ is known; the initial $P_0(z_{P0},\rho_{P0},\alpha_{P0})$, $Q_0(z_{Q0},\rho_{Q0},\alpha_{Q0})$, $\Delta\alpha_0\Delta z_0$, and φ_0 are given; the exact temporal location of P is determined by the desired MCR geometry, and $z_P(t)$ and $\rho_P(t)$ are to be found.

There is a "length law": if the fiber is fixed on P and P moves around the rod, the fiber length between P and A ($PA=AQ_0+\text{PQ}+QQ_0$) has to be constant. Then the change of QQ_0 and PQ must the same, because AQ_0 is constant, too.

$$d(QQ_0)+d(PQ)=-d(AQ_0)=0, \tag{5}$$

where

$$d(QQ_0)=\frac{\rho_Q d\alpha_Q}{\cos\varphi}=\frac{dz_Q}{\sin\varphi},$$

and

$$d(PQ)=d\frac{\sqrt{\rho_P^2-\rho_Q^2}}{\cos\varphi}=d(\Delta z^2+\rho_P^2-\rho_Q^2)^{1/2}\quad(\text{when }\varphi\neq 0).$$

Then we have

$$\frac{dz_Q}{\sin\varphi}=-d\frac{\sqrt{\rho_P^2-\rho_Q^2}}{\cos\varphi}\quad\text{or}\quad dz_Q=-\frac{\sin^2\varphi}{\cos^2\varphi}\sqrt{\rho_P^2-\rho_Q^2}d\varphi-\tan\varphi d\sqrt{\rho_P^2-\rho_Q^2}. \tag{6}$$

From Eq. (4):

$$dz_Q=dz_P-d\Delta z=dz_P-d(\tan\varphi\sqrt{\rho_P^2-\rho_Q^2})=dz_P-\tan\varphi d\sqrt{\rho_P^2-\rho_Q^2}-\sqrt{\rho_P^2-\rho_Q^2}/\cos^2\varphi d\varphi. \tag{7}$$

From Eq. (6), Eq. (7) becomes:

$$d\varphi=\frac{dz_P}{\sqrt{\rho_P^2-\rho_Q^2}}=\frac{\tan\varphi dz_P}{\Delta z}. \tag{8}$$

If φ is not constant, the first-order differential equation for z_P as function of φ gives:

$$dz_P-(z_P/\tan\varphi)d\varphi=-(z_Q/\tan\varphi)d\varphi. \tag{9}$$

The solution of which (z_P as function of φ) is

$$z_P(\varphi)=\sin\varphi\left(\int -z_Q(\varphi)\frac{\cos\varphi}{\sin^2\varphi}d\varphi+C\right),\quad C\text{ is constant}. \tag{10}$$

By substituting Eq. (10) into Eq. (8) and solving for ρ_P,

$$\rho_P(\varphi)=\{[z_P'(\varphi)]^2+\rho_Q^2(\varphi)\}^{1/2}. \tag{11}$$

By substituting Eq. (11) into Eq. (3) and solving for α_P:

$$\alpha_P(\varphi)=\alpha_Q(\varphi)+\Delta\alpha(\varphi)=\alpha_Q(\varphi)+\arccos\left(\frac{\rho_Q(\varphi)}{\rho_P(\varphi)}\right). \tag{12}$$

Since $\alpha_P(t)=\omega t+\alpha_{P0}$, $\varphi(t)$ can be derived from Eq. (12) and subsisted into Eqs. (10) and (11) to give $z_P(t)$ and $\rho_P(t)$.

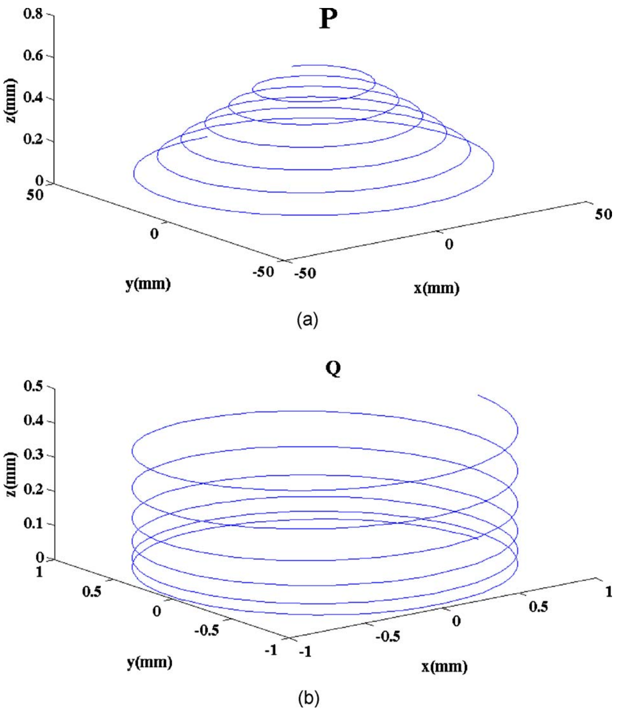

Fig. 2 Trajectories of P and Q for $z_Q=1/100(\alpha_Q/2\pi)^2+1/100(\alpha_Q/2\pi)$ (mm), assuming $z_{Q0}=0$, $z_{P0}=0$, $\rho_{P0}=50$ mm, and $\rho_{Q0}=1$ mm.

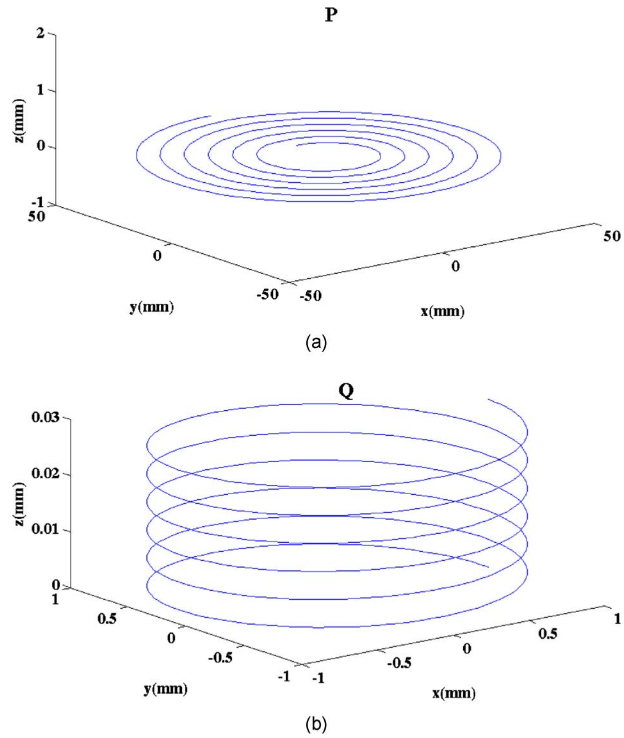

Fig. 3 Trajectories of P and Q for $z_Q=1/100\alpha_Q/4\pi^2$ (mm), $z_{Q0}=0$, $z_{P0}=0$, $\rho_{P0}=50$ mm, and $\rho_{Q0}=1$ mm, when φ is constant.

When φ is constant, from Eq. (8), $dz_P=0$, thus $z_P=z_{P0}$ is constant, but ρ_P is not constant, and in fact P will move to the rod in the plane perpendicular to the z axis, keeping $\Delta z/PQ$ constant. From Eqs. (1)–(4),

$$\rho_P=\left[\left(\frac{z_{P_0}-z_Q}{\tan\varphi}\right)^2+\rho_Q^2\right]^{1/2}, \tag{13}$$

$$\alpha_P=\alpha_Q+\arccos\left(\frac{\rho_Q}{\rho_P}\right)=\alpha_Q+\arccos\left(\frac{\rho_Q}{\left[\left(\frac{z_{P_0}-z_Q}{\tan\varphi}\right)^2+\rho_Q^2\right]^{1/2}}\right). \tag{14}$$

As an example, we calculate the movements necessary to fabricate an MCR with tapered spacing; the trajectory of Q is assumed to be:

$$z_Q=\frac{1}{100}\left(\frac{\alpha_Q}{2\pi}\right)^2+\frac{1}{100}\left(\frac{\alpha_Q}{2\pi}\right)\ (\text{mm}). \tag{15}$$

If the initial values are $z_{Q0}=0$, $z_{P0}=0$, $\rho_{P0}=50$ mm, and $\rho_{Q0}=1$ mm, the trajectory of P can be obtained from Eqs. (10)–(12). Figure 2 shows the locations of P and Q.

As another example, we also calculate the movements necessary to fabricate an MCR with uniform spacing when φ is assumed constant; the trajectory of Q is assumed to be:

$$z_Q=\frac{1}{100}\frac{\alpha_Q}{4\pi^2}.$$

The trajectory of P can be obtained from Eqs. (13) and (14). Figure 3 shows the trajectories of P and Q for this case.

3 Revised and Simplified Model

Although this method can be used to manufacture any kind of MCR profile, it poses great challenges because the stage velocity has to be controlled extremely well and the microfiber has to be kept tensioned: if the microfiber becomes loose, the approach angle between microfiber and support rod will change, while if it is too tight it can break.

In practical experiments, the microfiber is free to move at P; a V-groove and a small mass at the end of P can be used to maintain the direction and the tension of the microfiber; ρ_P is constant. This leads to:

$$\begin{cases}\alpha_P=\arccos\rho_Q/\rho_P\\ z_P=\sqrt{\rho_P^2-\rho_Q^2}\tan\varphi+z_Q\\ \tan\varphi=dz_Q/\rho_Q d\alpha_Q\end{cases}. \tag{16}$$

If both z_P and ρ_P are constant (simplest case), P moves with a rotation velocity ω, thus the location of Q is given by:

$$z_Q=z_P+C\exp\left(-\frac{\rho_Q(\alpha_Q-\alpha_{Q0})}{\sqrt{\rho_P^2-\rho_Q^2}}\right). \tag{17}$$

(C is constant.) If the initial values are $\alpha_{Q0}=0$ and $z_{Q0}=0$, since $\rho_P\gg\rho_Q$:

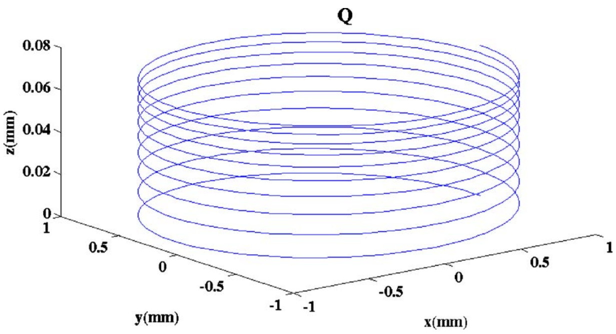

Fig. 4 The trajectory of Q for φ=arctan(0.1/50), z_P=0.1 mm, ρ_P=50 mm, and ρ_Q=1 mm.

$$z_Q \approx z_{P0}\left[1-\exp\left(-\frac{\rho_Q}{\rho_P}\alpha_Q\right)\right]. \tag{18}$$

As an example, we assume z_{P0}=0.1 mm, ρ_{P0}=50 mm, ρ_{Q0}=1 mm, and φ=arctan(0.1/50). The position of Q is shown in Fig. 4.

In theory, $z_Q \to z_P$ when $\alpha_Q \to \infty$, so the microfiber can always be wrapped very tightly. In practice, the taper length is limited; therefore the initial z_P has to be kept as small as possible.

Another issue arises from the fact that the rod axis is not the same as the z axis. If the angle between support rod and z axis is ψ, the microfiber will be wrapped on the rod with $\varphi=\psi$. To get an effective coupling between adjacent turns, the distance should be smaller than several micrometers, if we assume the maximum of the distance between adjacent turns to be 5 μm, so $\varphi < 0.0016 \sim 0.09^0$. It is such a small angle that this is a critical issue for practical manufacturing of MCRs.

4 Summary

In summary, the mathematical model for the manufacture of MCR with predesigned profiles is investigated and simplifications are proposed. The trajectory of microfiber pigtails is determined for arbitrary profiles of MCRs. It can be used to manufacture MCRs with arbitrary profiles.

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