Optics Communications 285 (2012) 5144–5147

Contents lists available at SciVerse ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Coupling influence on the refractive index sensitivity of photonic wire ring resonator

Wei Guo, Fei Xu*, Yan-qing Lu

National Laboratory of Solid State Microstructures and College of Engineering and Applied Sciences, Nanjing University, Nanjing 210093, PR China

ARTICLE INFO

Article history:
Received 10 June 2012
Received in revised form
23 July 2012
Accepted 3 August 2012
Available online 23 August 2012

Keywords:
Ring resonator sensor
Resonance condition
SOI

ABSTRACT

By taking the coupling effect into consideration, we study the resonance condition of a photonic wire microring resonator (PWRR) sensor and compare our results with the previous work. Simulation results show that the resonant wavelength and sensitivity strongly depend on the coupling strength. The difference caused by the coupling effect can be up to tens of nanometers for the resonant peak position and tens of nm/RIU for the sensitivity in a silicon-on-insulator (SOI) PWRR. Such a giant influence from coupling effect cannot be disregarded and should be considered seriously for the design and application of PWRRs. It also shows an alternative tuning technique by controlling the coupling strength.

1. Introduction

Optical-waveguide-based microring resonators are versatile wavelength-selective elements that can constitute cascading building blocks in large-scale integrated photonic circuits for use as lasers, add-drop filters, dispersion compensators and sensors [1–7]. Optical-waveguide-based microring resonator sensors have a lot of attractive properties including low cost, simple configuration, robustness, compactness, easy surface chemical modifications and high integration capability with electronic and photonic devices [8]. A basic microring resonator consists of a reentrant ring-shaped waveguide and a coupling region connecting the ring-shaped waveguide to the external circuitry, which usually contains two adjacent waveguides that are very close to each other. For most bio/chemical applications, the resonant light circulates along the ring resonator and an evanescent wave in the ring waveguide interacts with the analytes surrounding the waveguide. The cross-section geometry and effective index of the waveguide would have a strong effect on the sensitivity and a smaller cross-section is preferred because it would allow a higher evanescent field outside the waveguide for sensing. The resonant wavelength of a resonator is always considered to only depend on the effective index of the waveguide. However, in previous work on the analysis and design of resonators, the physical length of the coupling region is considered as zero and the weak coupling effect in the coupling region is assumed to make no contribution to the resonance condition and sensitivity and are always ignored.

Following the rapid development of fabrication technologies [9–14], low loss subwavelength-diameter photonic wire waveguides have emerged as an ideal sensor element and attracted more and more attention. In the past few years, microring resonator sensors based on various photonic wire waveguides have been widely implemented, including planar silica/silicon waveguides and even circular microfibers [2,4,6]. Due to the small size and large evanescent field, photonic wire microring resonator (PWRR) sensors offer a lot of advantages such as extremely compact size, high sensitivity, high selectivity and low detection limits. In a PWRR, the coupling in the coupling region can be very strong because of the large evanescent field, sufficient inter-waveguide coupling and relatively long coupling length. The strong coupling effect is possible to greatly influence the resonance condition and sensitivity and should not be ignored.

In this paper, we investigate the resonance condition and sensitivity by considering the strong coupling effect in a PWRR sensor. Our analysis and simulation show that the strong coupling effect makes a significant contribution to the resonant peak position and sensitivity. By tuning the coupling strength, the resonant peak position can be shifted as far as tens of nanometers and the sensitivity can be modified by as large as tens of nm/RIU in a silicon-on-insulator (SOI) PWRR. It is important and helpful for the design and applications of PWRR sensors. It also shows an alternative tuning technique by controlling the coupling strength. In this paper, we do not theoretically investigate the relationship between *Q*-factor and the strong coupling effect. *Q*-factor is related to the loss of the waveguide. As there is no additional loss caused by the strong coupling effect, we believe the *Q*-factor will not decrease obviously.

2. Theory analysis and numerical model

Fig. 1 shows a schematic of a typical PWRR which usually includes a coupling region where two pieces of wire waveguides

* Corresponding author.
E-mail addresses: feixu@nju.edu.cn (F. Xu), yqlu@nju.edu.cn (Y.-q. Lu).

0030-4018/$ - see front matter
http://dx.doi.org/10.1016/j.optcom.2012.08.005

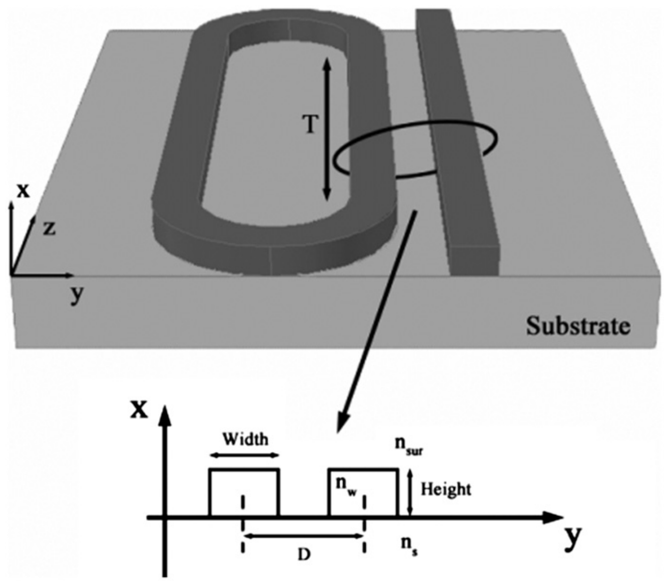

Fig. 1. Basic configuration of a PWRR and the cross-section of the coupling region.

are close to each other. We assume L to be the loop length and T to be the coupling length. The cross-section of the PWRR is also shown in Fig. 1. We set D to be the pitch between the two segments in the coupling region, n_w, n_s and n_{sur} to be the refractive indices of the waveguide, substrate and environment (or analyte), respectively.

We start from the coupled mode Eq. [15]:

$$\begin{cases} \frac{dA}{dz} + c_{12}\frac{dB}{dz} + j\chi_1 A + j\kappa_{12}B = 0 \\ \frac{dB}{dz} + c_{21}\frac{dA}{dz} + j\chi_2 B + j\kappa_{21}A = 0 \end{cases} \tag{1}$$

where

$$\begin{cases} \kappa_{pq} = \frac{\omega\varepsilon_0 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(N^2-N_q^2)E_p^* E_q dxdy}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u_z(E_p^* H_p + E_p H_p^*)dxdy} \\ c_{pq} = \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u_z(E_p^* H_q + E_q H_p^*)dxdy}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u_z(E_p^* H_p + E_p H_p^*)dxdy} \\ \chi_p = \frac{\omega\varepsilon_0 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(N^2-N_p^2)E_p^* E_p dxdy}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u_z(E_p^* H_p + E_p H_p^*)dxdy} \end{cases} \tag{2}$$

and the pair of p and q are either $(p, q)=(1, 2)$ or $(2, 1)$. A and B are the electromagnetic field amplitudes of the two segments. N and N_p are the refractive index distribution of the entire coupled segments and each segment, E_p and H_p ($p=1, 2$) are the electric and magnetic field of the eigen modes in each segment before mode coupling, κ is the coupling coefficient of the resonator, c describes the individual waveguide mode overlap and is called the butt coupling coefficient, χ represents the perturbation to the electromagnetic field distributions caused by the adjacent waveguide. We assume the two segments to be identical and we have some simplifications: $\kappa_{12}=\kappa_{21}=\kappa$, $c_{12}=c_{21}=c$ and $\chi_1=\chi_2=\chi$.

In previous work, c and χ are very small and often assumed to be zero, the physical length of the coupling region is also considered as zero, and then, the coupled mode equation is simplified as

$$\begin{cases} \frac{dA}{dz} + j\kappa B = 0 \\ \frac{dB}{dz} + j\kappa A = 0 \end{cases}. \tag{3}$$

That will lead to a traditional resonance condition [16]:

$$n_{eff}L = m\lambda_R \quad m = 1,2,3\ldots \tag{4}$$

where n_{eff} is the effective index, λ_R is the resonant wavelength, respectively. The resonant wavelength only depends on the effective index of the waveguide and is unrelated to the coupling effect.

However, if width of the waveguide and pitch D between the two segments is small enough, there will be large evanescent field and strong coupling effect. In order to investigate the resonator strictly, we take c and χ into consideration and by simplifying Eq. (1) we get

$$\begin{cases} \frac{\partial A}{\partial z} = jMA + jNB \\ \frac{\partial B}{\partial z} = jMB + jNA \end{cases}. \tag{5}$$

Here $M = \chi - \kappa c/c^2 - 1$ and $N = \kappa - c\chi/c^2 - 1$.

We separate A and B in Eq. (5) and find:

$$\begin{cases} \frac{\partial^2 A}{\partial z^2} - 2\frac{\partial A}{\partial z}jM - (M^2 - N^2)A = 0 \\ \frac{\partial^2 B}{\partial z^2} - 2\frac{\partial B}{\partial z}jM - (M^2 - N^2)B = 0 \end{cases} \tag{6}$$

and by solving Eq. (6) we get A and B:

$$\begin{cases} A(z) = C_1\exp[j(M+N)z] - C_2\exp[j(M-N)z] \\ B(z) = C_1\exp[j(M+N)z] + C_2\exp[j(M-N)z] \end{cases} \tag{7}$$

The coefficients C_1 and C_2 can be calculated by considering the boundary conditions:

$$\begin{cases} A(0) = 1 \\ B(0) = B(T)\exp(j\beta L) \end{cases} \tag{8}$$

and we get:

$$\begin{cases} C_1 = \frac{1 - \exp[j(\lambda_2 T + \beta L)]}{2 - \exp[j(\lambda_1 T + \beta L)] - \exp[j(\lambda_2 T + \beta L)]} \\ C_2 = \frac{\exp[j(\lambda_1 T + \beta L)] - 1}{2 - \exp[j(\lambda_1 T + \beta L)] - \exp[j(\lambda_2 T + \beta L)]} \end{cases} \tag{9}$$

where

$$\begin{cases} \lambda_1 = M + N \\ \lambda_2 = M - N \end{cases}.$$

Thus, we get the values of A and B with different coupling length. Considering the waveguide loss, the value of $A(T)$ should be minimum in the resonance condition where T is the coupling length in our model. Based on that condition we find the modified resonance condition as:

$$n_{eff}L + \lambda_R(MT/2\pi) = m\lambda_R \quad m = 1,2,3\ldots \tag{10}$$

Compared with the previous one, the coupling effect is included in the resonance condition with an added modified item $\lambda_R(MT/2\pi)$. So next we calculate the values of M to evaluate the importance of the coupling effect.

3. Simulation results

In our simulation, a full vector finite element method is used to calculate the distribution of the electromagnetic field of one waveguide and then, we use Matlab to calculate the coupling effect of the resonator based on the equations we mentioned above. We consider one proto typical system: a PWRR based on rectangular $1-\text{to}-1.5$ aspect ratio embedded SOI waveguides. However, our theory and results are also suitable for other PWRRs such as microfiber resonators.

All the simulations are based on the fundamental mode and the high order modes are not taken into consideration because the coupling effect makes a significant influence when the size of the waveguide is relatively small and for those parameters, the waveguide is usually single-mode. We calculate a long range of the structure parameters to observe how the coupling effect

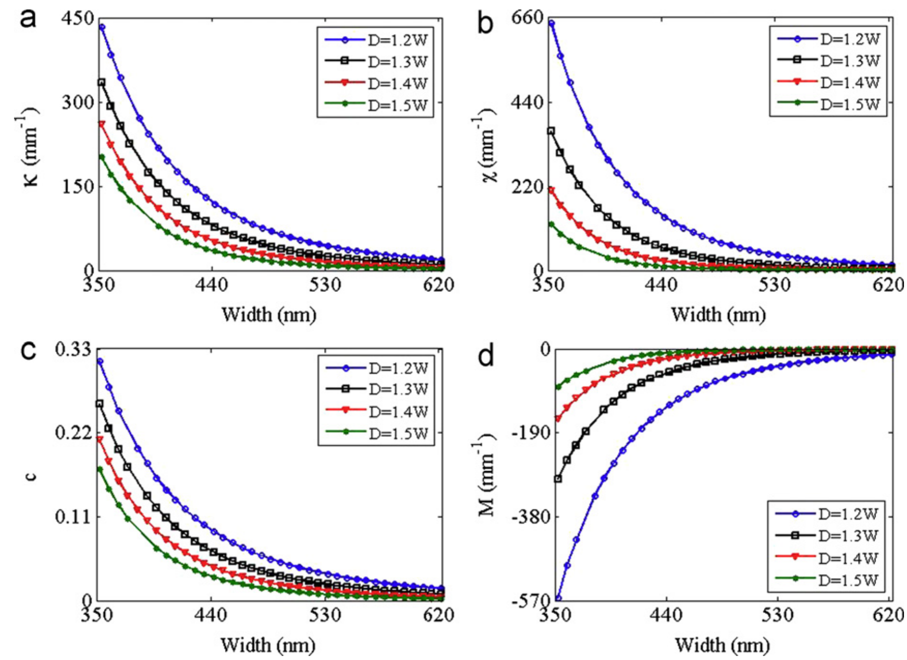

Fig. 2. The calculated (a) κ, (b) c, (c) χ and (d) M profiles of a resonator with different widthWand different pitch D.

changes with the increase of the waveguide size. Besides, we believe taking the high order modes into account will lead to similar results.

We calculate the values of κ, c, χ and M with different width W and pitch D in Fig. 2. Here we assume $n_{\mathrm{sur}}=1.34$ for water. As we set $W=1.5\,H$, they increase by the same ratio. According to our simulation results, the values of κ, c and χ decrease as width increases from 350 nm to 620 nm. This is because the intensity of the evanescent field decays with the increase of the waveguide size. As we have mentioned above, the values of c and χ are often assumed to be zero and that will lead to $M=0$, according to $M=\chi-\kappa c/c^2-1$. However, as we can see from Fig. 2(c), the values of M can be as large as $-564\ \mathrm{mm}^{-1}$ for $W=353$ nm and $D=1.2$ W. Thus, the strong coupling effect cannot be ignored especially when the structure parameters of the waveguide are relatively small.

As the resonance condition is modified as: $n_{\mathrm{eff}}L+\lambda_{\mathrm{R}}(MT/2\pi)=m\lambda_{\mathrm{R}}$. Compared with the previous one ($n_{\mathrm{eff}}L=m\lambda_{\mathrm{R}}$), the resonant wavelength is modified to $\Delta\lambda_{\mathrm{R}}=\lambda_{\mathrm{R}}(M/\beta)\alpha$, which is the influence of the coupling effect. In addition, it also leads to the resonant peak position difference between the traditional formula and our theory. Here $\alpha=T/L$ $(0<\alpha\leq 1)$ is the ratio of coupling length to ring length. In a planar PWRR, the common value range is $0<\alpha\leq 0.25$, but it is also possible that $\alpha>0.25$. In an all-coupling microfiber coil resonator, $\alpha=1$.

For weak coupling, the values of c and χ are very small and the values of M are close to zero according to $M=\chi-\kappa c/c^2-1$. Then, $\Delta\lambda_{\mathrm{R}}$ approaches zero as it is proportional to M. For strong coupling, we calculate the values of c and χ and get M and $\Delta\lambda_{\mathrm{R}}$ according to the equations mentioned above. In Fig. 3 we plot the values of $\Delta\lambda_{\mathrm{R}}$ at different W and D. As the size of the waveguide becomes smaller, the coupling-effect contribution to resonant wavelength $\Delta\lambda_{\mathrm{R}}$ gets larger and if $\alpha=0.2$ ($\alpha=1$) the resonant peak position difference can be as large as -20.6 nm (-103 nm) at $W=353$ nm and $D=1.2$ W. Such a large difference means that the coupling greatly influences the resonant peak position and should be taken into account when designing and analyzing a PWRR. It also shows a possible method to tune the resonant wavelength by the control of pitch D.

The sensitivity of a PWRR is usually defined by the shift of the resonant wavelength as a function of the refractive index change of the surrounding medium

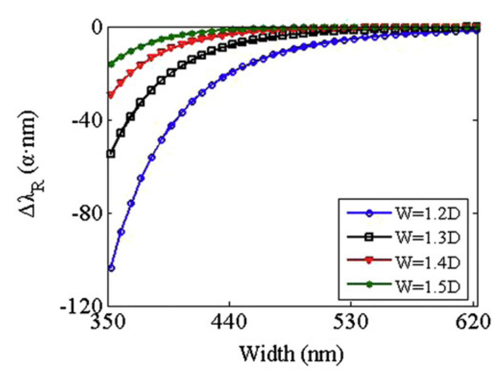

Fig. 3. The calculated $\Delta\lambda_{\mathrm{R}}$ profiles of a PWRR with different width W and different pitch D.

$$S=\frac{\partial\lambda_{\mathrm{R}}}{\partial n_{\mathrm{sur}}} \tag{11}$$

According to the resonance condition $n_{\mathrm{eff}}L+\lambda_{\mathrm{R}}(MT/2\pi)=m\lambda_{\mathrm{R}}$, we get from the coupled mode equation, we have

$$S=\frac{\partial\lambda_{\mathrm{R}}}{\partial n_{\mathrm{sur}}}=S_{\mathrm{n}}+S_{\mathrm{k}} \tag{12}$$

where

$$\begin{cases} S_{\mathrm{n}}=\frac{2\pi}{\beta}\frac{\partial n_{\mathrm{eff}}}{\partial nE} \\ S_{\mathrm{k}}=\alpha\frac{\lambda}{\beta}\frac{\partial M}{\partial nE} \end{cases} \tag{13}$$

S_{n} and S_{k} are the contributions of the evanescent field and the coupling effect, respectively.

In traditional theory of previous literature, only S_{n} is considered. We plot S_{n} and S_{k} profiles at different parameters in Fig. 4. Here we assume $n_{\mathrm{sur}}=1.34$. We can see from Fig. 4(a) and (b) that with the increase of the waveguide size, S_{k} and S_{n} both drop. This is because the intensity of the evanescent field and the coupling between the two segments both decay. However, if the waveguide size is small enough, the values of S_k make a relatively high

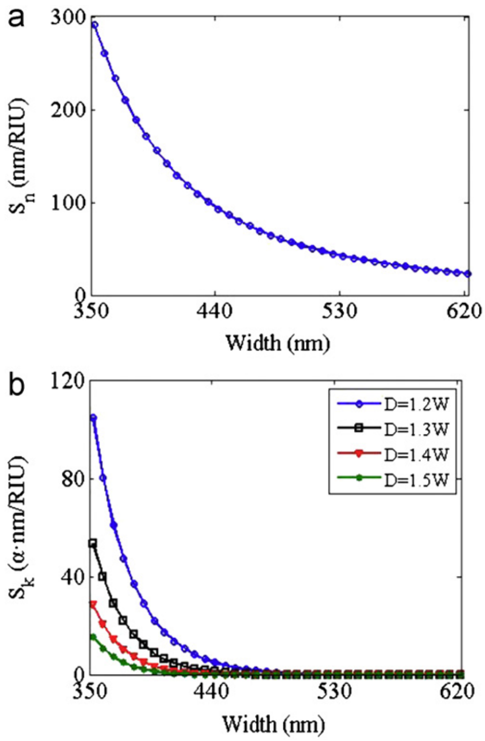

Fig. 4. The calculated (a) S_n and (b) S_k profiles of a resonator with different width *W* and different pitch *D*.

contribution to the total sensitivity of the resonator. For example, at $W=353$ nm and $D=1.2$ W, S_k can be as large as 20.8 nm/RIU (104 nm/RIU) if $\alpha=0.2$ ($\alpha=1$). Such a high S_k means that the coupling strength greatly influences the sensitivity and should be taken into account when designing and analyzing a PWRR. It also shows an alternative tuning technique by controlling the pitch.

4. Discussion and conclusion

In conclusion, we study the resonance condition and sensitivity of a PWRR by considering the coupling effect. Deriving the full coupled wave equations, we modify the previous theory on resonance condition and sensitivity. Our analysis and simulation show that the resonant wavelength and sensitivity strongly depend on the coupling effect, which is always disregarded in previous work. In a typical SOI PWRR, the contribution and influence can be as high as tens of nanometers on the resonant wavelength and tens of nm/RIU on the sensitivity, respectively. Our calculation shows that such a giant influence from coupling effect cannot be disregarded and should be considered seriously for the design and application of PWRRs. In addition, future work can also focus on finding such a structure where S_n and S_k have different signs and in that case the resonator can be immune to external perturbations.

Acknowledgment

This work was supported by the National 973 program under contract nos. 2011CBA00200, 2012CB921803 and 2010CB327800 and the NSFC program under contract nos. 11074117 and 60977039. The authors also acknowledge the support from PAPD and the Fundamental Research Funds for the Central Universities.

References

[1] Y. Sun, X. Fan, Analytical and Bioanalytical Chemistry399 (2011) 205.
[2] K. De Vos, I. Bartolozzi, E. Schacht, P. Bienstman, R. Baets, Optics Express 15 (2007) 7610.
[3] C. Stamatiadis, K. Vyrsokinos, L. Stampoulidis, I. Lazarou, A. Maziotis, J. Bolten, M. Karl, T. Wahlbrink, P. De Heyn, Z. Sheng, D. Van Thourhout, H. Avramopoulos, Journal of Lightwave Technology 29 (2011) 3054.
[4] A. Ksendzov, Y. Lin, Optics Letters 30 (2005) 3344.
[5] F. Xu, Q. Wang, J.-F Zhou, W. Hu, Y.-Q Lu, IEEE Journal of Selected Topics in Quantum Electronics 17 (2011) 1102.
[6] F. Xu, V. Pruneri, V. Finazzi, G. Brambilla, Optics Express 16 (2008) 1062.
[7] Y.-F. Xiao, V. Gaddam, L. Yang, Optics Express 16 (2008) 12538.
[8] C.Y. Chao, L.J. Guo, Journal of Lightwave Technology 24 (2006) 1395.
[9] G. Brambilla, V. Finazzi, D.J. Richardson, Optics Express 12 (2004) 2258.
[10] L.M. Tong, R.R. Gattass, J.B. Ashcom, S.L. He, J.Y. Lou, M.Y. Shen, I. Maxwell, E. Mazur, Nature 426 (2003) 816.
[11] L.M. Tong, J.Y. Lou, R.R. Gattass, S.L. He, X.W. Chen, L. Liu, E. Mazur, Nano Letters. 5 (2005) 259.
[12] R. Yan, D. Gargas, P. Yang, Nature Photonics 3 (2009) 569.
[13] P. Prabhathan, V.M. Murukeshan, Z. Jing, P.V. Ramana, Optics Express 17 (2009) 15330.
[14] Y.A. Vlasov, S.J. McNab, Optics Express 12 (2004) 1622.
[15] K. Okamoto, Fundamentals of optical waveguides, second ed., Elsevier, Amsterdam; Boston, 2006.
[16] K.S. Lim, A.A. Jasim, S.S.A. Damanhuri, S.W. Harun, B.M.A. Rahman, H. Ahmad, Applied Optics 50 (2011) 5912–5916.