From the above analysis, the differences between the normal QPM technique and the QPM in ring could be found. The diagram of these two types of QPM is shown in Fig. 1(b). The QPM technique in a waveguide needs periodically alternating ferroelectric domain structures [10, 11]. The light experiences positive and negative nonlinear coefficients one by one. In a QPM ring, light propagates along the circular waveguide and goes through two reversely poled areas in each loop, which means the light still can see different domains periodically. It is just similar to the case in a straight QPM material. In another word, the necessary periodicity needed to achieve QPM process comes from a virtual unfolded straight waveguide rather than a practical periodic domain structure. From this point of view, a partly domain inverted LN ring with radius R is equivalent to a periodically poled waveguide with period 2πR. However, in ring system, due to the geometry difference, the momentum conversion law would change to angular momentum conversion law.

3. Simulation results and discussions

Since the periodically poled lithium niobate (PPLN) is the most popular QPM material, let's also select LN to study the QPM in a two-dimensional micro ring. As the simplest process, SHG is studied at first. From Fig. 1(a), a ring-shaped LN is partially poled forming two domains with opposite spontaneous polarizations. A coupling waveguide is placed close to the ring. The fundamental wave (FW) is pumped from the left port of the waveguide to generate the second-harmonic wave (SHW) in the ring. A wavelength splitter and light detectors could be installed at the right port to measure the powers of FW and its SHW, respectively. We define θ as the opening angle of the domain inverted region. If $\theta = \pi$, the crystal is equally divided into two domains. To simulate the SHG in the ring, we assume the coupling coefficient between the input waveguide and the ring resonator is the same for both FW and SHW.

Because nonlinear optical conversion happens inside the ring, both the FW and SHW should be right at the cavity's WGMs. Since each WGM satisfy a specific wavelength, both the FW wavelength and the SHW wavelength should be right at the WGM wavelength. This condition could be satisfied though proper ring parameter design with some matching wavelengths. It is even tunable through adjusting the operation temperature or applying an electric field. In our simulation, we set the fundamental mode at 1539.5 nm and the second harmonic mode at 769.7 nm, corresponding to two WGMs of a 2D LN ring with internal radius of 12.74 μm and width of 1 μm. When calculating the WGMs, both the material and waveguide dispersions are considered. The operation temperature is set at 25°C and index of air cladding is 1. The index of LN is obtained from the Sellmeier equation [19] that is temperature sensitive. If errors occur in accuracy of the radius or width, the WGMs of the ring may have an offset from the design. The pumping wavelength thus should be finely adjusted to fit the WGMs. However, we may also change the temperature to tune the WGMs back in principle.

Before studying the SHG process, one factor should be noticed. Due to the character of this so-called "add-drop" system [17], whether a quadratic or a third-order nonlinear ring, only part of the input power is able to couple into the ring and participate in frequency conversion. Based on this, the internal and external SHG efficiency of this system could be defined. The external SHG efficiency is the ratio between the output SHW power and the input pump power into the bus waveguide. And the internal SHG efficiency is defined as the output SHW power over the portion of the FW power coupled into the ring. Obviously, the internal efficiency should be always higher than the external one.

To derive the SHG internal and external efficiencies, Eqs. (3) and (4) should be used. We assume a stable state of the system, so the light field amplitude does not change with the time. This makes the formulations change to quadratic equations. The internal and external SHG efficiencies versus the pump power in the input bus waveguide thus could be easily obtained as shown in Figs. 2(a) and 2(b). Coupling coefficients τ = 0.1, 0.2, 0.3 have been simulated, corresponding Q-factors around 10^3~10^4, which are quite achievable. On the other hand, the ratio of internal efficiency to external efficiency is $\tau^2/2$. This is because the proportion of

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power coupled into the ring to the input pump power is $\tau^2/2$. Due to the nature of nonlinear effect, the SHG efficiencies all rise with the increase of pump power. They gradually saturate when the internal efficiency approaches 1. And, in this case, the external efficiency is close to $\tau^2/2$ as shown with the black solid curve in Fig. 2(b). For $\tau = 0.1$, $\tau^2/2$ gives the highest limit of 5% for external efficiency.

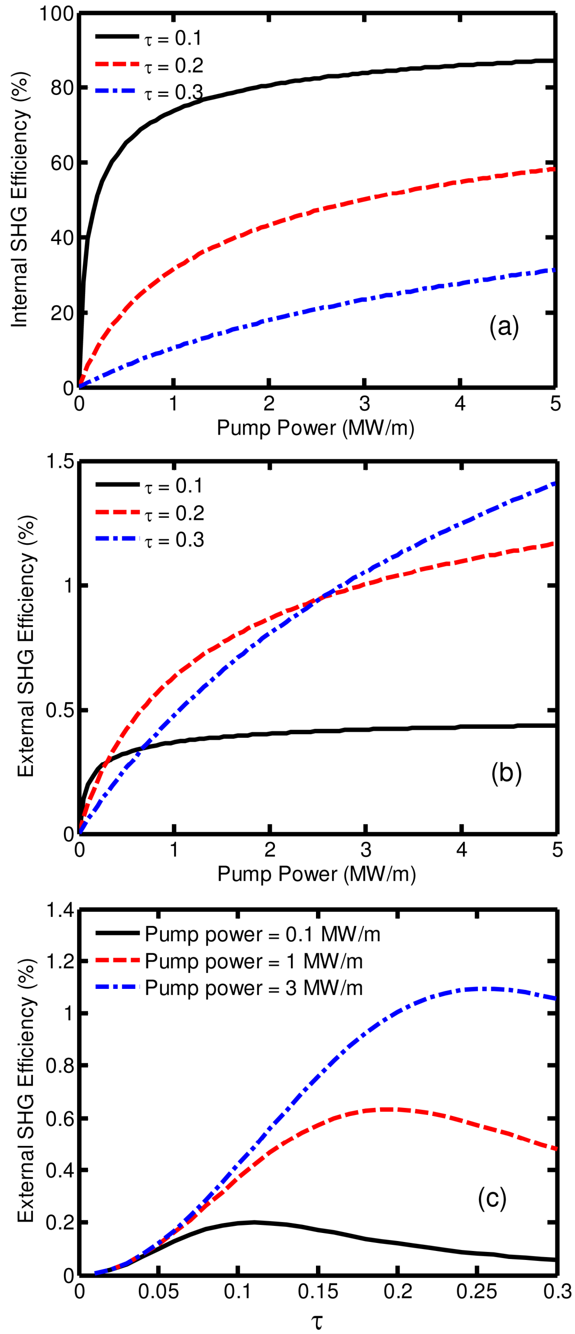

Fig. 2. The (a) internal and (b) external SHG efficiencies with different pump power, where the solid, dashed and dash-dot curves correspond to $\tau = 0.1, 0.2, 0.3$, respectively; (c) The external SHG efficiencies with different τ, where the solid, dashed and dash-dot curves correspond to pump power = 0.1, 1, 3 MW/m.

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In addition to the efficiency limit, other interesting phenomena could be found from the figures. Because of the reduction of light intensity with the increase of τ, the internal SHG efficiency simply reduces when τ increases. But for the external SHG efficiency, things are different. Although the internal efficiency still decreases, the internal to external efficiency ratio $\tau^2/2$ increases at the same time. That makes the external efficiency reach a maximum at a given τ value, as shown in Fig. 2(c). However, the efficiency peaks emerge at different τ values for different pump powers. This feature should be considered when design such a nonlinear ring system.

If we compare the QPM SHG in the ring with normal QPM in a bulk or waveguide PPLN, the characteristics shows both similarities and differences. Light in the ring also experiences different nonlinear coefficients, but the domain inverted area should match the involved cavity modes. The external SHG efficiency also rises with the increase of pump power, but it cannot go beyond a limit at $\tau^2/2$. In addition, the efficiency shows complicate relationship with the coupling coefficient τ. If the pump power is high enough, bigger coupling coefficient gives rise to larger external SHG efficiency, while in lower pump power cases, the internal SHG efficiency would not reaches a maximum, the external SHG efficiency will not simply rise with the increase of τ. The QPM in a nonlinear ring really has many interesting properties that worth studying.

Only two WGMs are considered in the SHG calculations above. However, there could be many modes in this ring system. In optical frequency comb generation processes [1], cascading SHG and OPG process should be considered with more WGMs involved. In comparison with a straight waveguide [4], the ring system makes the cascading nonlinear processes much more complicated. The involved lights should be right at the ring cavity's WGMs. We assume the pump light at a mode frequency of ω_p and generate a pair of modes at ω_m and ω_n with $2\omega_p = \omega_{sh} = \omega_m + \omega_n$. The SHG process could be described as the system absorbs two photons with the frequency ω_p and generate the photon at second harmonic mode with frequency $\omega_{sh} = 2\omega_p$. Then the OPG process could be described as the system absorbs the photon at the SH mode and generates a photon pair with different frequencies ω_m and ω_n. Due to the dispersion of the ring, this condition could not be severely satisfied. However, if the line shapes of the ring resonances are considered, then the generated SHW and frequency down-converted signal and idler waves can be made to fit within a specific mode-spacing. Furthermore, if there is a frequency spacing mismatch between the WGMs and the signals generated via SHG and spontaneous down-conversion, there should be a finite bandwidth over which the generated comb can fit in the ring's resonances. So as an approximation, the modes that satisfy the condition $\Delta\omega_{sh} > \omega_{sh} - \omega_m - \omega_n$ are selected. $\Delta\omega_{sh}$ represents the full width of the maximum of the SH mode. It depends on the Q-factor and the coupling condition of our system. We set τ at 0.01 and $\Delta\omega_{sh} = 0.1$ THz for the simulation. In order to include more comb modes, the wavelength of pump changes to 1556.7 nm, thus a SHW at 778.35 nm is obtained. In this case, there would be 22 WGMs included in the calculation, including the pump mode, the SH mode and 20 modes with the resonating wavelength around the pump mode. The mode number of these modes are p ± 1, 2, 3, ..., 10. Here 'p' indicates the mode number of pump light that is equal to 111. The mode number of SHW WGM is 234. As a consequence, $\Delta = 12$ and the open angle of the reversely poled domain is set at $\pi/4$. If error occurs for the open angle, the nonlinear conversion efficiency may be affected. The tolerance could be obtained through calculating the angular part integration of Eq. (6). With a 5° offset, the coupling coefficient κ_{pnm} would only be reduced by 1.5%. The pump power is set at 10 MW/cm.

Based on these conditions, the calculation similar to the previous one could be made. The coupling among these 22 modes is described by Eqs. (3) and (4). The stable state assumption makes the equations change to a group of equations. Then the power of each mode could be produced through solving the equations. Based on the Q-factor and power of each WGM, the output spectra are obtained as shown in Figs. 3(a) and 3(b).

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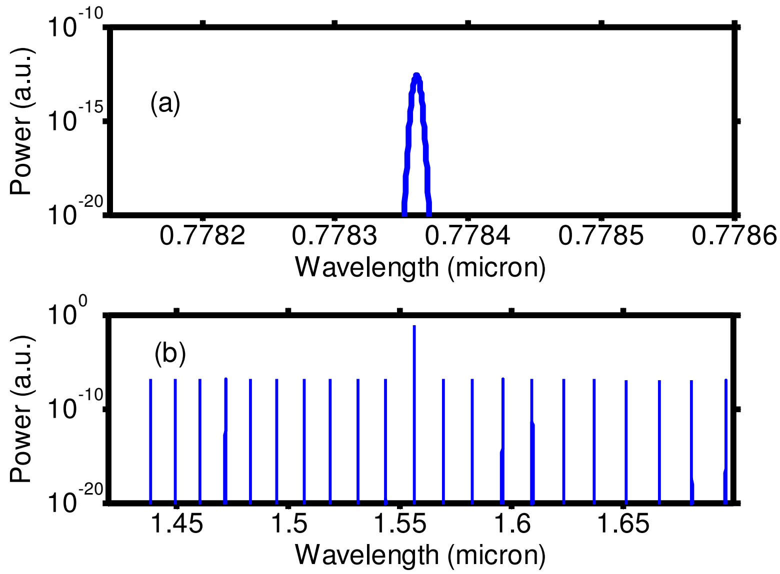

Fig. 3. The output spectra of (a) the second harmonic mode and (b) the pump mode and the comb-like modes generated from the second harmonic mode.

Figure 3(a) shows the output spectra of SH mode through QPM. The output power of this mode is 0.31 W/cm. Compare with the pure SHG, it is a much smaller value. This is due to the couplings between second harmonic mode and the other 20 modes around the pump mode. The power of the second harmonic mode is further transferred to these 20 modes with the help of OPG processes.

Figure 3(b) shows the output spectra of pump mode and the comb modes that are generated from QPM OPG. The typical line-width of the comb modes is still around 0.1 THz. The output power of the pump mode is 9.999 MW/cm. It is approximately equal to the input pump power. This is because the coupling coefficient is set to a relatively low value 0.01. And it leads to the 500 W/cm input power from waveguide to ring. The other part of the power will just propagate through the waveguide without the participation of nonlinear process. According to the simulation results, each of the 20 degenerated modes has an output power around 14 W/cm. So the total power of these modes would be 280 W/cm; the internal efficiency of the 20 modes generation is 56%, which is a quite high value, means efficient optical frequency comb generation really may be achieved in such a nonlinear ring. Adjusting the pump power, different internal efficiencies also could be obtained.

In addition, it's worth mentioning that there should be tens parametric processes that satisfy the QPM in the ring simultaneously. Although it looks very difficult in a traditional PPLN, the WGM nature of the LN ring makes the processes happen. According to Eqs. (6)-(8) and the related discussions, as long as $\Delta = m + n - p \neq 0$. The involved WGMs (m, n and p) always could be quasi-phase-matched given a suitable domain inverted open angle θ. For an odd Δ, $\theta = \pi$, while for an even $\Delta = 2^n \times odd$, $\theta = \pi/2^n$. In the comb generation process, p corresponds to the mode number of SHW, while m and n are for a pair of parametric down-conversion modes, respectively. Assume m ≥ n. If modes m and n are generated, modes m + 1 and n-1 should be generated simultaneously through QPM. This is because there is no change for $\Delta = m + n - p = (m + 1) + (n-1) - p$. For the same reason, modes m + k and n-k are also obtainable. A serial of parametric down-conversion processes thus could be realized together, which results a frequency comb at different WGMs.

Considering from the system level, our optical frequency comb shares a similar regime with the traditional third-order nonlinearity ones, as both systems are typical 'add-drop'

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systems. The overall comb generation rate thus should be limited by the waveguide-to-ring coupling coefficient. However, as long as the pump light is coupled in the ring, the internal efficiency could reach a very high level due to the intrinsically more efficient quadratic nonlinear optical process. In addition, although the dispersion between FWs and SH has been compensated through QPM, the dispersion among the FWs still limits the number of participant modes. In our proposal, a simple 2D system is investigated to better display the mechanism of QPM comb generation. However, although it is quite simplified with respect to a 3D counterpart, they have basically very similar characteristics in ring QPM and comb generation. Both of them are based on the nonlinear interaction among WGMs rather than straightforwardly propagating lights. For a 3D system, as longs as the FW and SHW are at the WGMs, QPM SHG is able to achieve just like in a 2D ring. In this case, the 2D and 3D systems are equivalent in physics and nonlinear optical behaviors.

To implement this idea, we may take a patterned poled LN according to our design with a suitable open angle. Then we may make a waveguide layer through proton-exchange that is mature in industry. To obtain a nice ring and input/output bus, femtosecond laser may be used to machine it precisely. The final 3D LN ring waveguide still has air cladding at top and two sides, while LN with slightly lower index acts as the bottom cladding. Obviously, the coupling and WGMs simulations would be much more complicated than a 2D system. FEM/FDTD methods might be used. However, it also gives the feasibility to further design the waveguide for dispersion compensation between different comb modes [2]. Through geometry optimization and careful fabrication, a broad band comb generation is expected.

4. Conclusion

In summary, the QPM technique is applied to a ring system. The general three wave coupling processes are studied theoretically. We found that a partly poled nonlinear ring could be just equivalent to a periodically poled QPM material. However, depending on the involved WGMs, the required poling areas are different. As an example, the SHG of a LN ring is investigated. The overall and internal SHG efficiencies are studied with various parameters, including the pump power, coupling coefficient and WGMs. Based on these results, an optical frequency comb solution is proposed that is from the cascading SHG and OPG processes in the ring. Higher comb generation efficiency is expected.

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