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Bistable Behavior in the Phase-Output Relation in Two-Dimensional Nonlinear Optical Superlattices

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With two waves incident on a two-dimensional (2D) nonlinear superlattice, the relation between ψ and the output intensities is bistable. Here, the ψ is the relative phase between the two waves. That is to say, as ψ is slowly varied within one cycle, a hysteresis loop in the ψ-output relation can be traced out. A very low threshold power is required in an appropriate 2D nonlinear superlattice for the optical bistability.

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In the recent years, nonlinear responses in intensity-dependent dielectric superlattices and multilayers have received much attention for their potential applications in optical communications and computing. In one-dimensional cases, some interesting phenomena, such as power bistability and frequency bistability [1-3], limiting [4], self-pulsing and chaos [5,6], and soliton [7], have been discovered. These phenomena are related to the transition between forbidden transmission states (FTS) and the allowed transmission states (ATS). Very recently, the studies of the nonlinear responses of the twodimensional (2D) superlattices, i.e., with their refractive index periodically modulated in two-dimensions, are beginning. In the linear cases, near the Bragg condition, the light transmission strongly depends upon the index-modulation (IM) strengths. Through the Kerr-form nonlinearity, the interference in the transmission field will perturb the IM strengths. Thus a positive feedback is formed. Through this new type of feedback mechanism, bistability, self-pulsing and chaos, have been discovered [8-10]. In the former paper of the present authors [11], the bistability related to the transition between a FTS and an ATS was demonstrated in the case of two incident waves. However, the q-output relation has not been studied, where ψ is the relative phase between the two incident waves. In this letter, we show that, as the ψ is slowly varied, a hysteresis loop in the ψ-output relation is traced out. We believe that this type of optical bistability, will be of benefit to the applications of 2D nonlinear superlattices in integrated optical devices.

Our theoretical model is applied to an isotropic lossless Kerr-form nonlinear dielectric superlattice. The linear periodically modulated refractive index is written in the form

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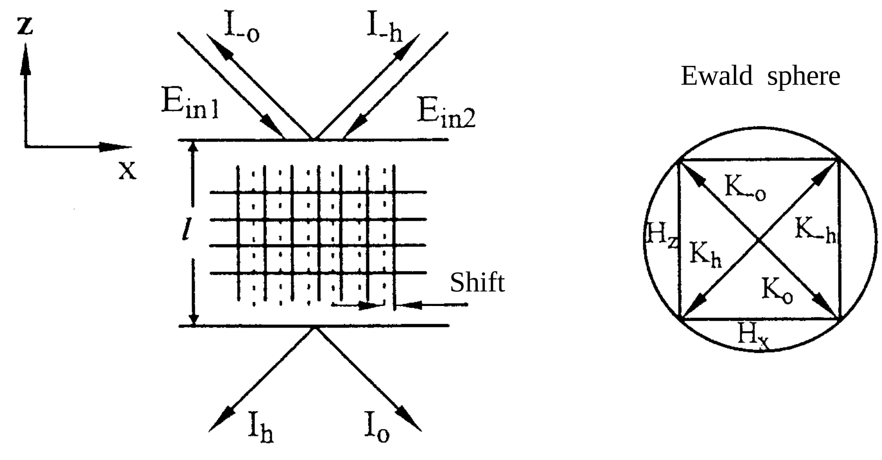


(a) (b)

FIG. 1. Four-wave diffraction in the two-dimensional periodic superlattice. (a) Schematic diagram of four-wave diffraction in real space. The dotted lines denote the incidence-dependent IM along x-direction formed by the incident fields E_{in1} and E_{in2}. (b) Bragg condition with four reciprocal points located on the Ewald sphere.

$$n = n_o \text{ t } n_z \cos(H_z z) + n_x \cos(H_x x), \tag{1}$$

where n_z and n_x are IM strengths along z and x direction, respectively, and satisfy $n_z, n_x \ll n_o$; H_z and N_x denote the periodicity in reciprocal space. Two coherent incident waves E_{in1} and E_{in2}, with the same incident Bragg angle, propagate symmetrically down the 2D superlattices [see Fig. l(a)]. The wave vectors satisfy the exact Bragg condition that four reciprocal points are located on the Ewald sphere. Thus four Bloch diffracted waves will be excited in the medium [see Fig. l(b)]. When the exact Bragg condition is satisfied, the field in the medium can be written as a sum of two forward and two backward diffracted waves

$$\begin{aligned} B(z) = \ & E,(z) \ \exp(i\mathbf{K}_o \cdot r) + E_h(z)\exp(i\mathbf{K}_h \cdot r) + E_{-o}(z)\exp(-i\mathbf{K}_o \cdot r) \\ & + E_{-h}(z)\exp(-i\mathbf{K}_h \cdot r), \end{aligned} \tag{2}$$

where $K_o = K_h = k_m$. k_m is the *average wave* number in the medium. The kerr-form nonlinearity can be described by the nonlinear polarization term

$$P_{NL}(r) = n_o n_\alpha |E|^2 E/4\pi, \tag{3}$$

where n_α is the nonlinear index. Inserting Eqs. (1)–(3) into Msxwell's wave equations, we obtain under the Bragg condition and in the slowly varying envelope approximation

$$\frac{dE_o}{dz} = -i\frac{k_m}{4\cos\theta_B}[2\delta n_o E_o + (M_x \text{ t } \delta n_x)E_h + \delta n_{x-z}E_{-o} + (M_z \text{ t } \delta n_z^*)E_{-h}]; \tag{4a}$$

$$\frac{dE_h}{dz} = -i\frac{k_m}{4\cos\theta_B}[(m_x+\delta n_x^*)E_o+2\delta n_o E_h+(m_z+\delta n_z^*)E_{-o}+\delta n_{x+y}^* E_{-h}]; \quad (4b)$$

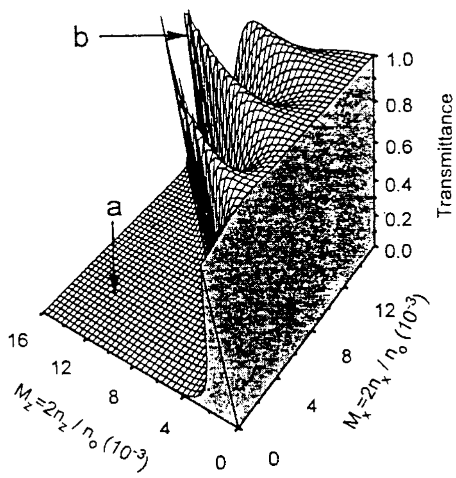
$$\frac{dE_{-o}}{dz} = -i\frac{k_m}{4\cos\theta_B}[\delta n_{x-z}^* E_o+(M_z+\delta n_z)E_h+2\delta n_o E_{-o}+(M_x+\delta n_x^*)E_{-h}]; \quad (4c)$$

$$\frac{dE_h}{dz} = -i\frac{k_m}{4\cos\theta_B}[(m_z+\delta n^z)E_o+\delta n_{x+z}E_h + (m_x + \delta n_x)E_{-o}+2\delta n_o E_{-h}]; \quad (4d)$$

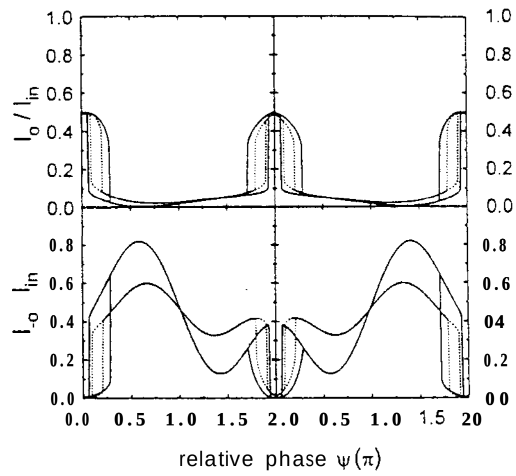
where $M_z = 2n_z/n_o, M_x = 2n_x/n_o, \delta n_o = \alpha(|E_o|^2 + |E_h|^2 + |E_{-o}|^2 + |E_{-h}|^2)$, $\delta n_z = 2\alpha(E_h^* E_{-o} + E_o^* E_{-h})$, $\delta n_x = 2\alpha(E_o E_h^* + E_{-o}^* E_{-h})$, $\delta n_{x+z} = 2\alpha(E_h^* E_{-h})$, $\delta n_{x-z} = 2\alpha(E_o E_{-o}^*)$, and θ_B is the Bragg angle. Here, α is defined as n_α/n_o. The boundary conditions of the Eqs. (4a)-(4d) are $E_o(0) = E_{in1}$, $E_h(0) = E_{in2}$, $E_{-o}(-l) = 0$, $E_{-h}(-l) = 0$. For convenience, we define the parameter γ as $|E_{in1}|^2/|E_{in2}|^2$, and I_{in} as $|E_{in1}|^2+|E_{in2}|^2$. I_{in} is the total incident intensity.

In terms of the linear results of Ref. [12], when $n_z > n_x$, the wave vectors of Bloch waves have imaginary part so that the Bloch waves are evanescent. This is very similar to the situation in which a wave located in the forbidden gap propagates down an 1D superlattice, and then these are so-called forbidden light transmission. On the contrary, when $n_z < n_x$, the wave vectors of Bloch waves are real, and then the Bloch waves will propagate through the medium unimpeded. These are so-called allowed light transmission. The IM-transmittance relation in the linear case is plotted in Fig.. 2. The transmittance is defined as $(I_o+I_h)/I_{in}$. There exist two distinct regions obviously in the figure. One is tbe region in which $n_z > n_x$. The amplitudes of the Bloch waves decay exponentially with propagation distance into the medium, and then the transmittance is very weak. These are related to the FTS. The other is the region in which $n_z < n_x$. The amplitudes of the Bloch waves will oscillate with the IM strengths. These are related to the ATS. For convenience, we define the parameter m as n_x/n_z. Thus the case of m < 1 is related to the FTS, and that of m > 1 is related to the ATS.

In the presence of two incident waves, the interference formed by two waves in the medium is characterized by a spatially periodic variation of the intensity. Through Kerr-form response nonlinearity, as shown in Fig. 1(a), the incidence-dependent periodic IM along x-direction are constructed by the two waves, where its periodicity is characterized by H_x in reciprocal space and its strength is proportional to the incident intensities. In the figure, the dotted lines denote the incidence-dependent IM along x-direction. This kind of process is similar to that of volume grating formation. It is usual that there exist a shift between the incidence-dependent IM and the preconstructed IM [namely the dotted lines and the solid lines in Fig. 1(a) are not overlapped]. The total IM strength along the x-direction, and then the effective m, will vary with the shift. When the shift is zero, the incidence-dependent IM matches the preconstructed IM, and then the effective m will reach its maximum at a certain incident intensity. On the contrary, when the shift reaches its maximum, the effective m will reach its minimum. Since the incidence-dependent IM is formed by the two incident waves, the shift is determined by the relative phase ψ between the two waves. In the case of $\psi = 0$, the shift is zero, and then the effective m reaches its maximum. In the case of $\psi=\pi$, the shift reaches its maximum, and then the effective m reaches its minimum. With the ψ scanning between 0 and 2π, the effective m will vary



(Fig. 2)



(Fig. 3)

FIG. 2. The relation between transmittance and index-modulation strengths under the exact Bragg condition. There are two distinct regions. Region *a* is related to the forbidden transmission state. Region *b is* related to the allowed transmission state. The structure parameters $k_m l/\cos\theta_B = 2.7 \times 10^3$.

FIG. 3. Optical phase bistable behavior in the ψ-output relations obtained by step-input numerical solution. The structure parameters $k_m l/\cos\theta_B = 2.7 \times 10^3$. $M_z = 5 \times 10^{-3}$, $M_x = 4.7 \times 10^{-3}$, $\gamma = 1$, and $I_{in} = 0.15$ for the solid line. $k_m l/\cos\theta_B = 2.7 \times 10^3$. $M_z = 6 \times 10^{-3}$, $M_x = 6.2 \times 10^{-3}$, $7 = 1$, and $I_{in} = 0.07$ for the dotted line. I_{in} is measured in I_{unit}, with $\alpha I_{unit} = 10^{-3}$.

between its minimum $m_{\min}$ and its maximum $m_{\max}$. It should be mentioned that $m_{\max}$. is proportional to the incident intensities, while $m_{\min}$ is inversely proportional to them. When the total incident intensity exceeds the threshold, $m_{\min} < 1$ and $m_{\max} > 1$. That is to say, when the ψ is varied within one cycle; for a 2D nonlinear optical superlattice, the effective m can pass across the transition point m = 1. Thus a transition between the FTS and the ATS will occur. For a 2D optical superlattice with its m near 1, the threshold for the transition should be low. We expect that the ψ-output relation is bistable through this transition.

Because of the complexity of these equations, Eqs. (4) can be only solved by numerical methods. We use the step-input method to integrate the Eqs. (4). The curves of output power versus the relative phase ψ in 2D nonlinear superlattices plotted in Fig. 3, where the linear m < 1 for the solid line and the linear m > 1 for the dotted line. In the figure, the low transmission is related to the FTS, namely the effective m < 1; the high transmission is related to the ATS, namely the effective m > 1. It can be seen that the hysteresis loop can be obtained through the variation of the ψ. Thus when the incident intensity exceeds the threshold, the $-output relation is bistable in a 2D nonlinear superlattice with its linear m larger than 1 or less than 1. For a 2D optical superlattice with its m near 1, the ψ bistable threshold can be very low. In the example as shown in Fig. 3, the bistable threshold is less than 0.07 in the case of $m > 1$ and is less than 0.15 in case of $m < 1$.

In conclusion, we have numerically demonstrated that, in the presence of two incident waves, the $-output relation is bistable through the IM mechanism. The threshold power for the bistability can be very low. This type of optical bistability might be useful to construct some special 2D optical bistable devices and phase-control switch.

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References

[1] V. M. Agranovich, S. A, Kiselev, and D. L. Mills, Phys. Rev. B44, 10917 (1991)
[2] J. Dancbert *et* al., Phys. Rev. B44, 8214 (1991).
[3] J. He and M. Cada, IEEE J. Quantum Electron 27, 1182 (1991).
[4] M. Scalore *et al.*, Phys. Rev. Lett. 73, 1368 (1994).
[5] H. G. Winful and G. D. Cooperman, Appl. Phys. Lett. 40, 298 (1982).
[6] H. G. Winful, R. Zamir, and S. Feldman, Appl. Phys. Lett. 58, 1001 (1982).
[7] W. Chen and D. L. Mills, Phys. Rev. Lett. 58, 160 (1987).
[8] B. Xu and N. B. Ming, Phys. Rev. Lett. 71, 1003 (1993).
[9] B. Xu and N. B. Ming, Phys. Rev. Lett. 71, 3959 (1993).
[10] B. Xu and N. B. Ming, Phys. Rev. A50, 5197 (1994).
[11] X. F. Chen et al., Appl. Phys. Lett. 67, 3538 (1995).
[12] J Feng and N. B. Ming, Phys. Rev. A40, 7047 (1989).